時系列解析（８）

－状態空間モデル－

配布用

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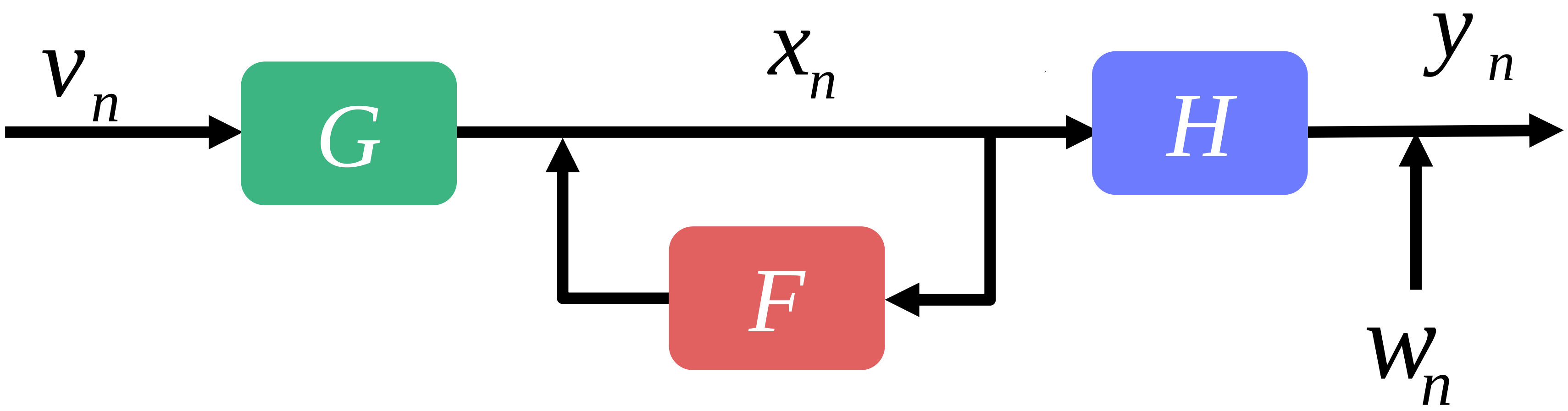
状態空間モデル

$$x_n = F_n x_{n-1} + G_n v_n$$ システムモデル

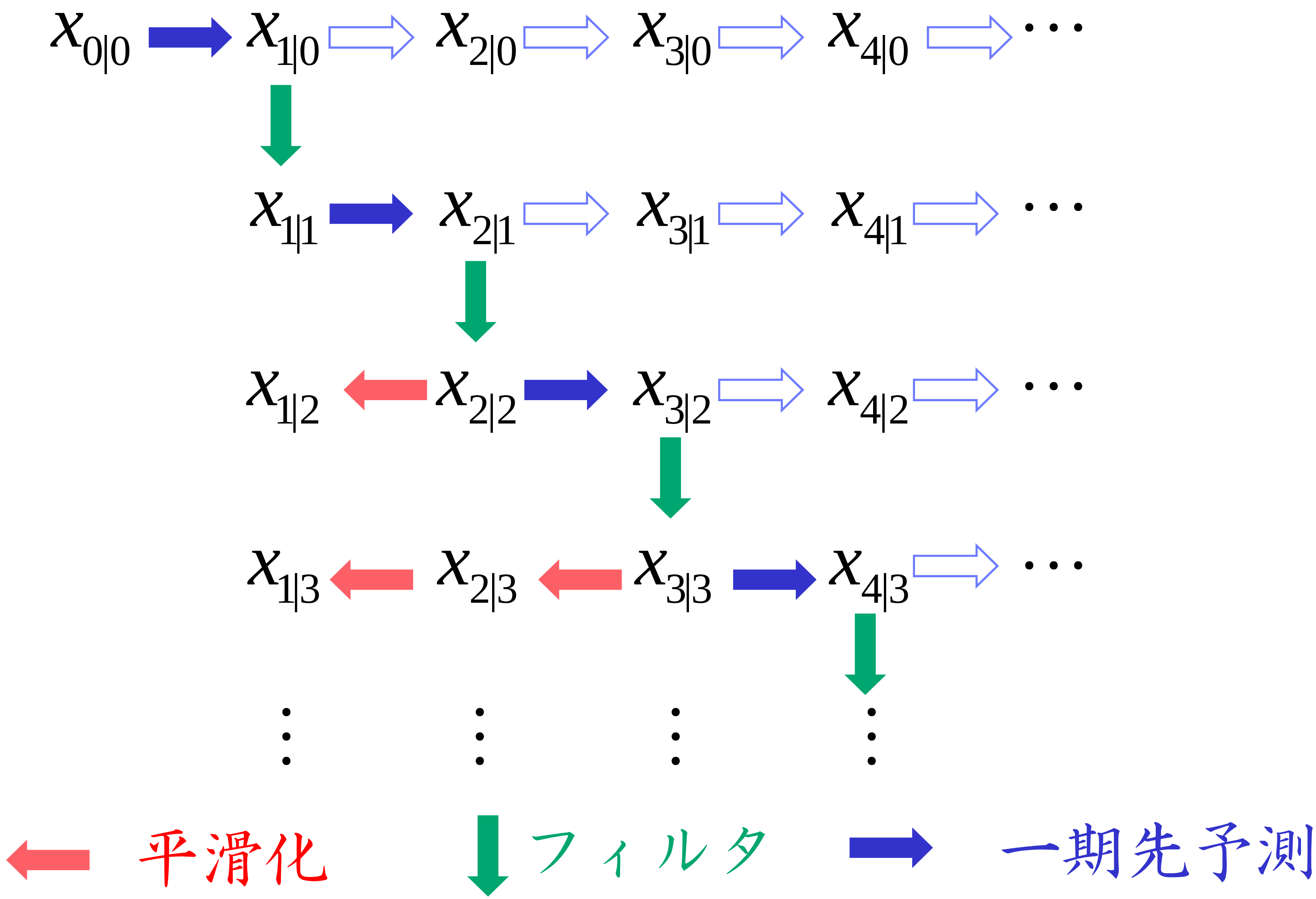
$$y_n = H_n x_n + w_n$$ 観測モデル

y_n	時系列	v_n	システムノイズ
x_n	状態ベクトル	w_n	観測ノイズ

$$v_n \sim N(0, Q_n), \qquad w_n \sim N(0, R_n)$$

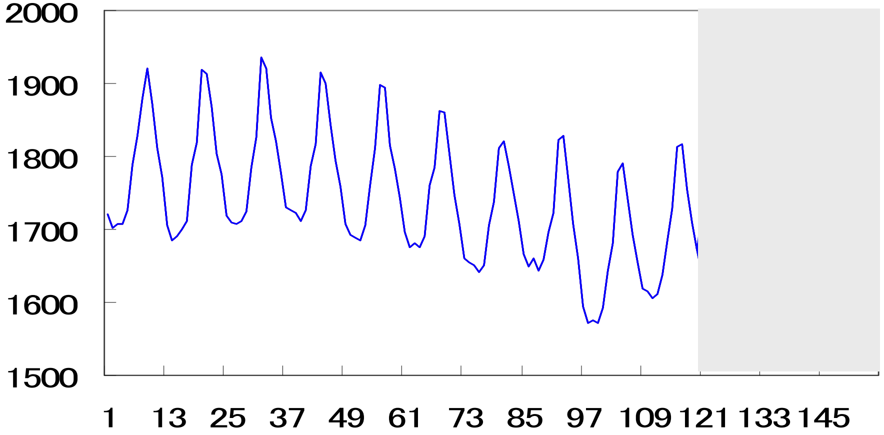


カルマンフィルタと平滑化

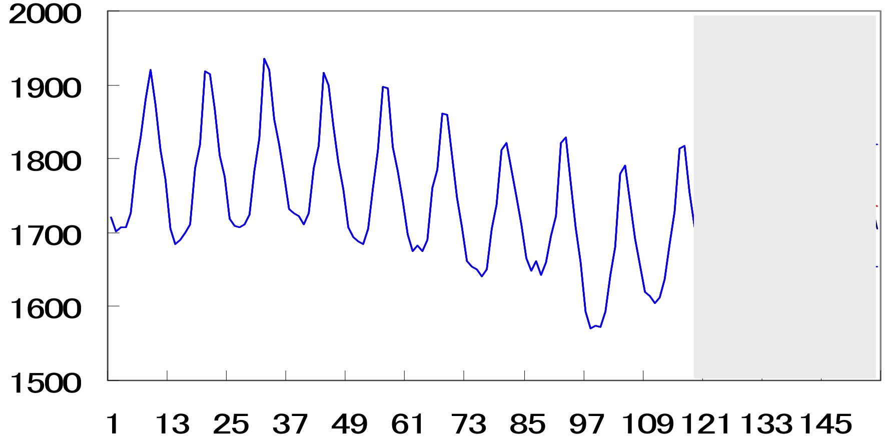


時系列の長期予測

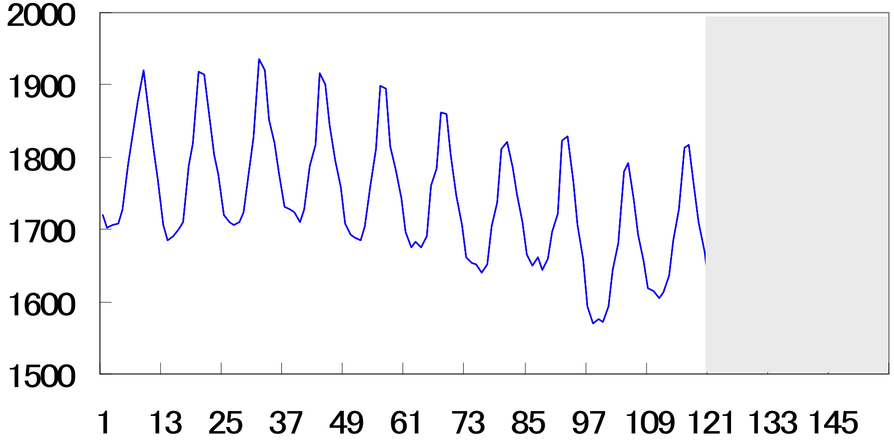
AR1



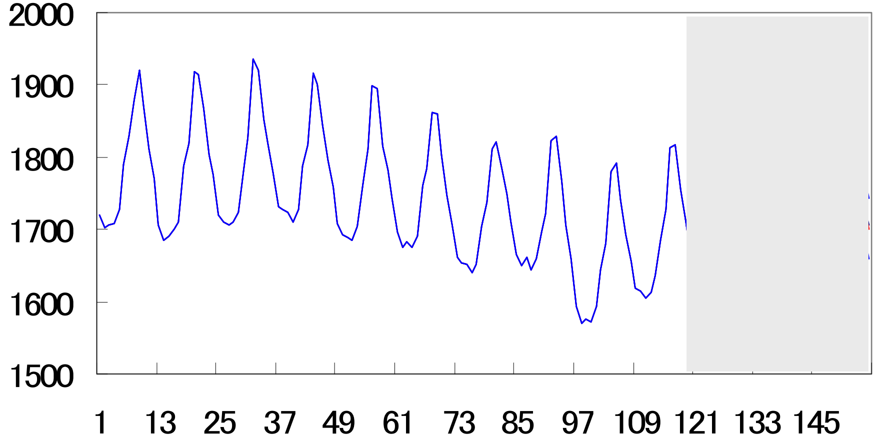
AR5



AR10

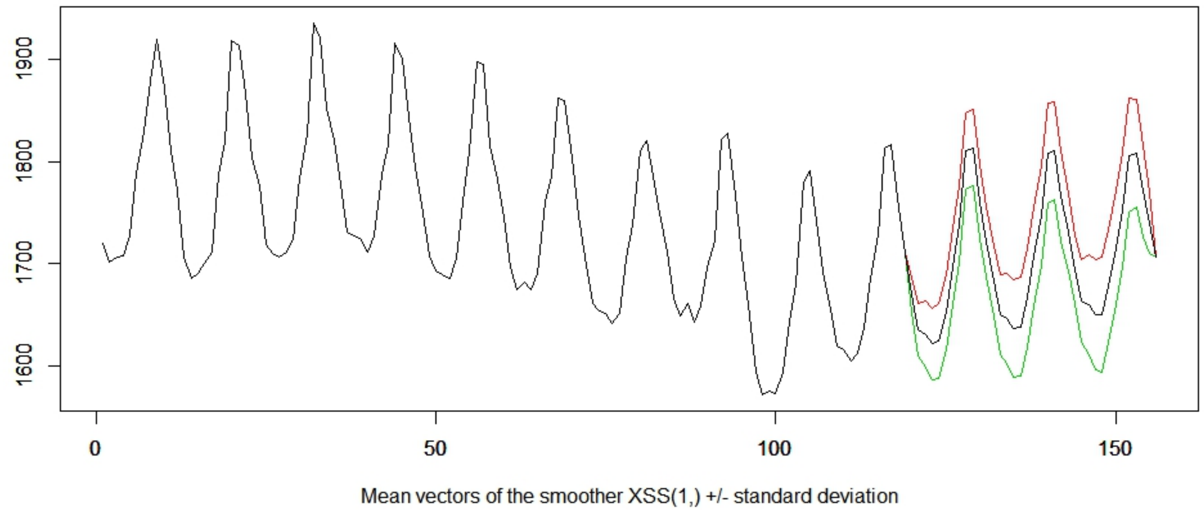


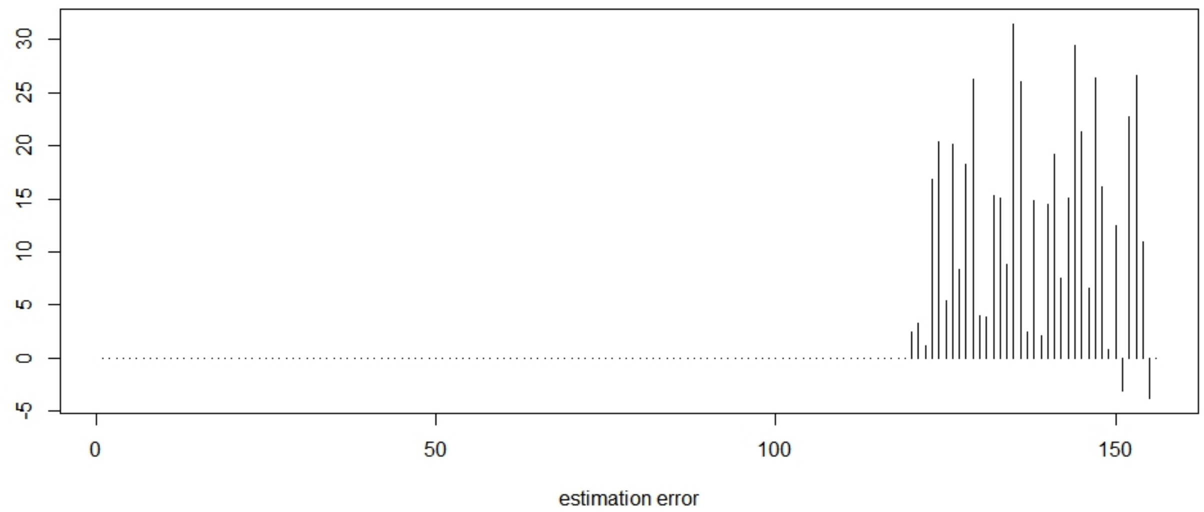
AR13



Rによる長期予測（AR次数：自動）

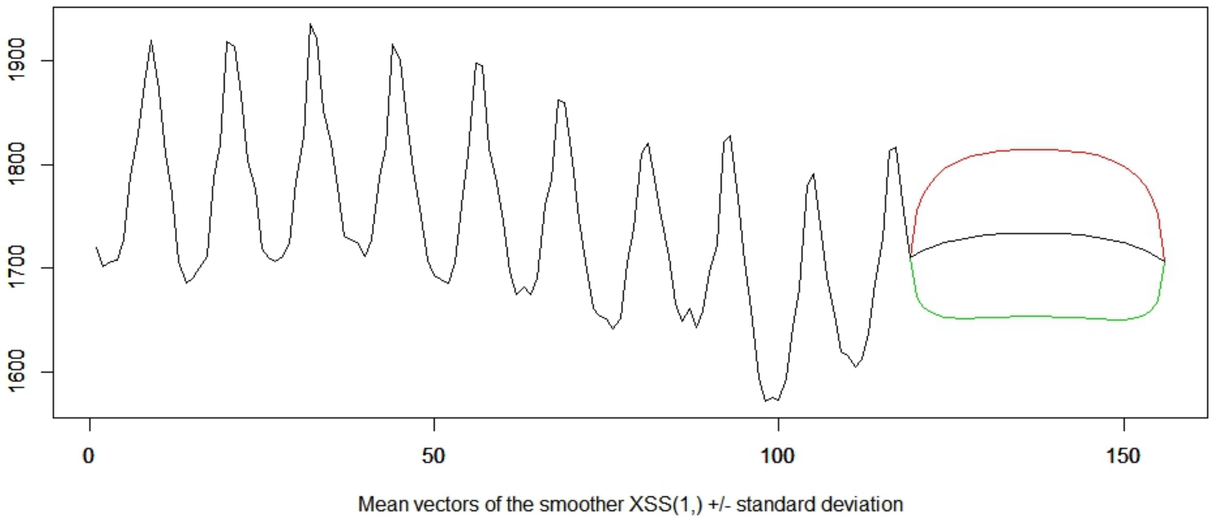
```
## AR model (l=1, k=1) 長期予測
#
  data(Blsallfood)
  z1 <- arfit(Blsallfood, plot=FALSE)
  m <- z1$maice.order
  tau2 <- z1$sigma2[m+1]
  arcoef <- z1$arcoef

  f <- matrix(0.0e0, m, m)
  f[1,] <- arcoef
  for( i in 2:m ) f[i,i-1] <- 1
  g <- c(1, rep(0.0e0, m-1))
  h <- c(1, rep(0.0e0, m-1))
  q <- tau2
  r <- 0.0e0
  x0 <- rep(0.0e0, m)
  v0 <- NULL

  z <- tsmooth(Blsallfood, f, g, h, q, r, x0, v0, 156, 156,
            missed=c(120), np=c(36))
# plot mean vector and estimation error
  xss <- z$mean.smooth[1,] + mean(Blsallfood)
  cov <- z$cov.smooth
  c1 <- xss + sqrt(cov[1,])
  c2 <- xss - sqrt(cov[1,])
  err <- z$esterr
  par(mfcol=c(2,1))
  ymax <- as.integer(max(xss,c1,c2)+1)
  ymin <- as.integer(min(xss,c1,c2)-1)
  plot(c1, type='l', ylim=c(ymin,ymax), col=2,
     xlab="Mean vectors of the smoother XSS(1,) +/- standard
deviation",  ylab="")

  par(new=TRUE)
  plot(c2, type='l', ylim=c(ymin,ymax), col=3, xlab="", ylab="")

  par(new=TRUE)
  plot(xss, type='l', ylim=c(ymin,ymax), xlab="", ylab="")
  plot(err[,1,1], type='h', xlim=c(1,length(xss)),
xlab="estimation error",  ylab="")
```

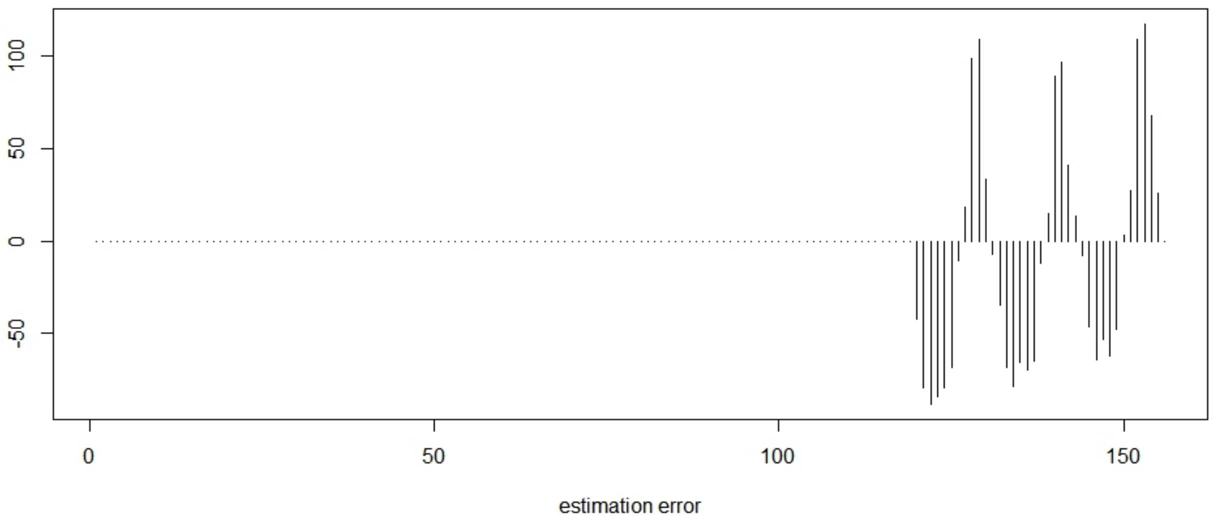




Rによる長期予測（AR次数＝１）

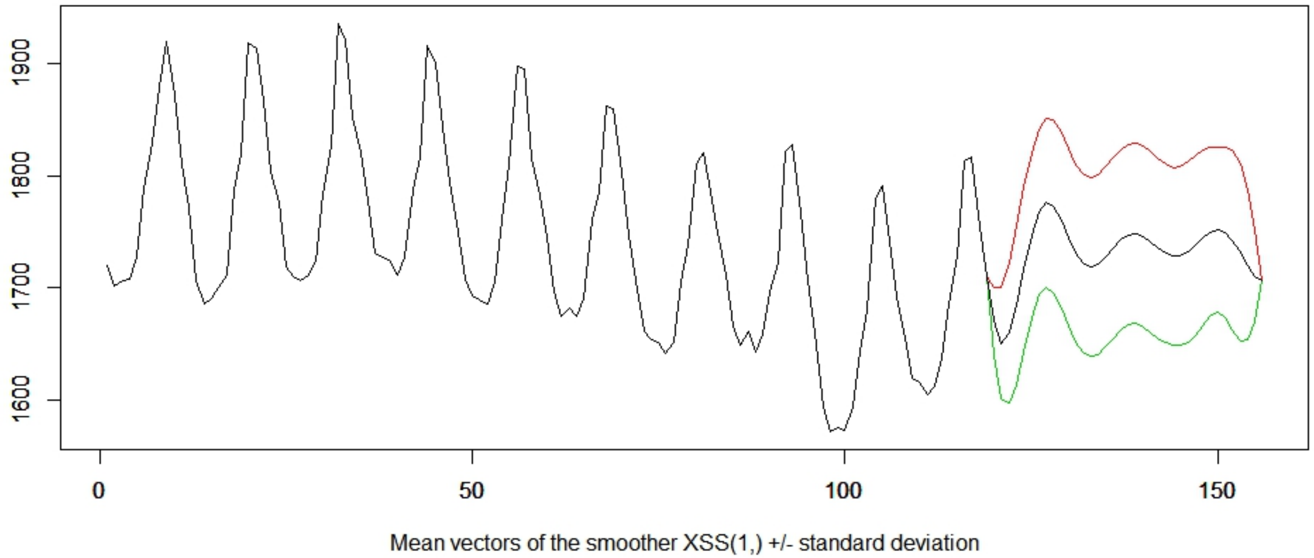
```
#  AR order=1
  z1 <- arfit(Blsallfood, plot=FALSE,lag=1)
  m <- z1$maice.order
  tau2 <- z1$sigma2[m+1]
  arcoef <- z1$arcoef
```

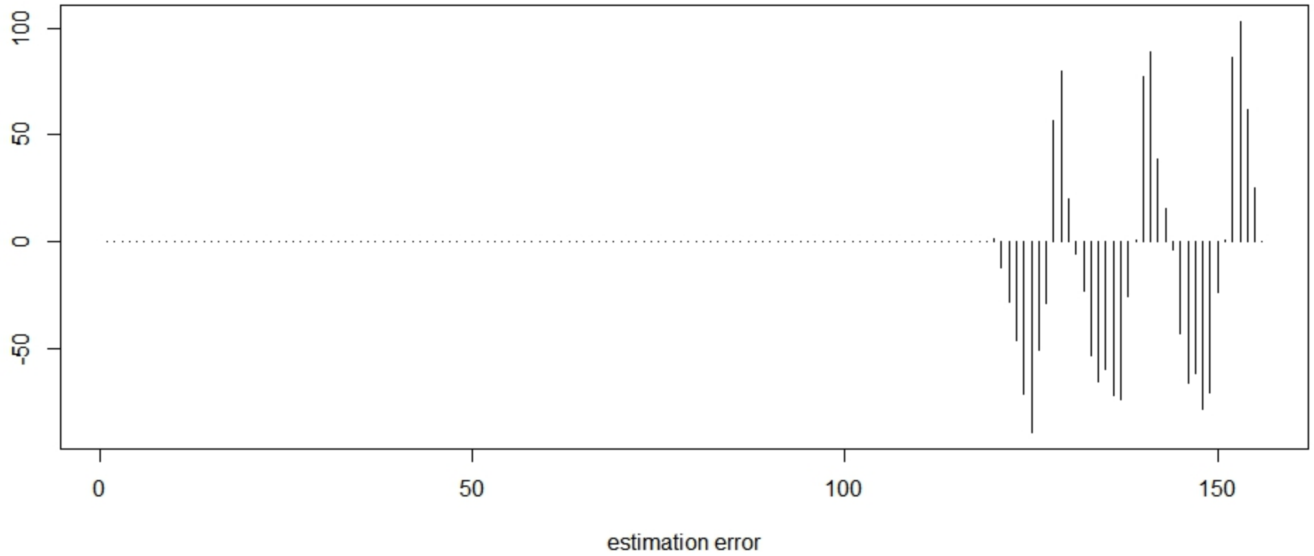




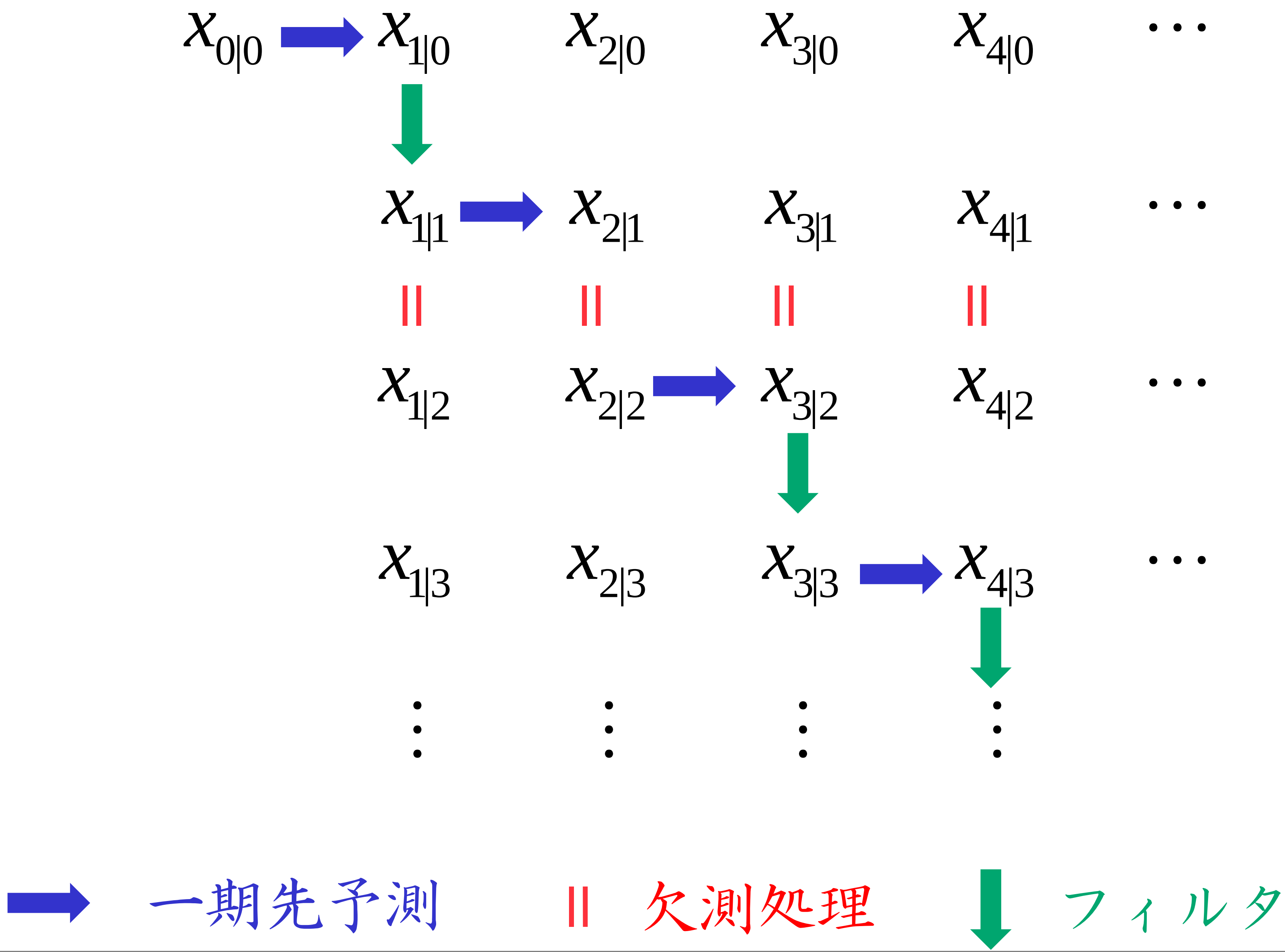
Rによる長期予測（AR次数＝5）

```
#  AR order=5
  z1 <- arfit(Blsallfood, plot=FALSE,lag=5)
  m <- z1$maice.order
  tau2 <- z1$sigma2[m+1]
  arcoef <- z1$arcoef
```





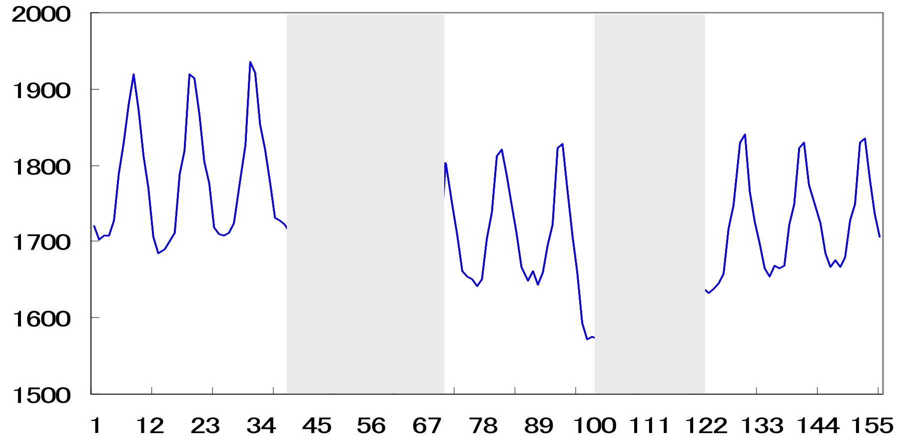
欠測値の処理



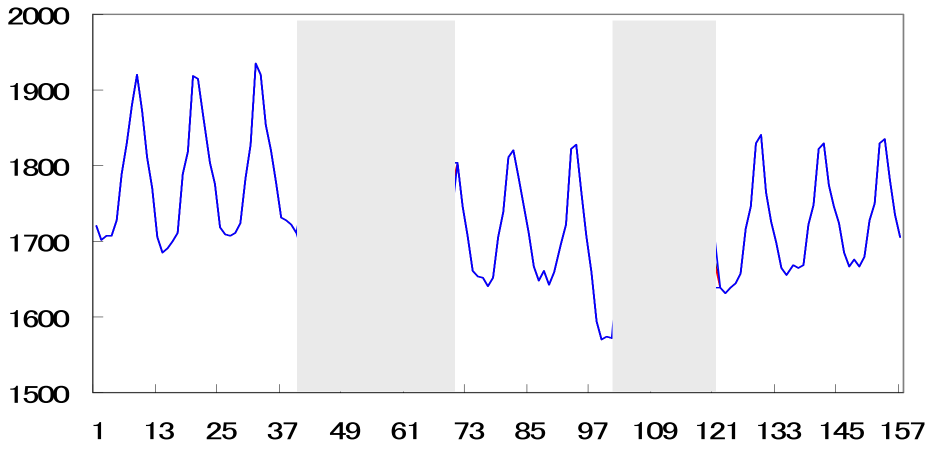
$x_{0|0}$ $x_{1|0}$ $x_{2|0}$ $x_{3|0}$ $x_{4|0}$ …
$x_{1|1}$ $x_{2|1}$ $x_{3|1}$ $x_{4|1}$ …
$x_{1|2}$ $x_{2|2}$ $x_{3|2}$ $x_{4|2}$ …
$x_{1|3}$ $x_{2|3}$ $x_{3|3}$ $x_{4|3}$ …
一期先予測
欠測処理
フィルタ

欠測値の補間

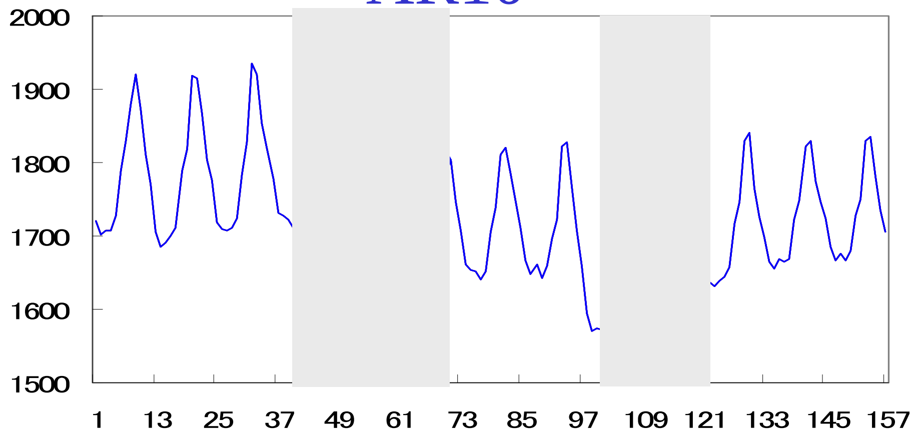
AR1



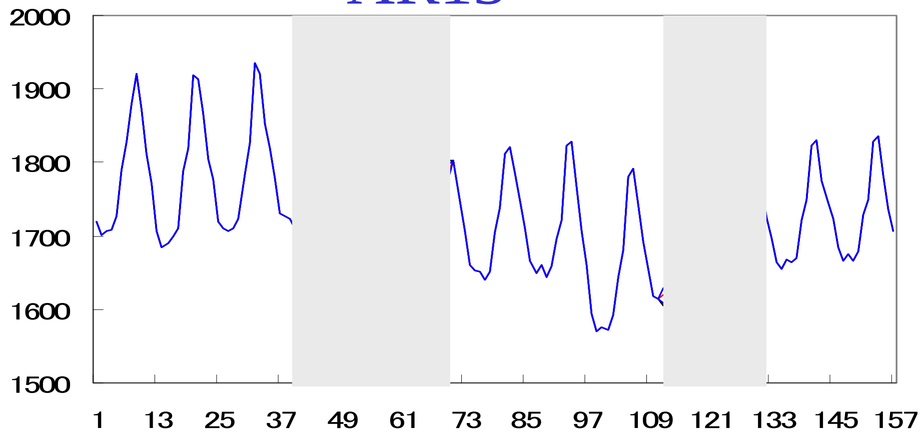
AR5



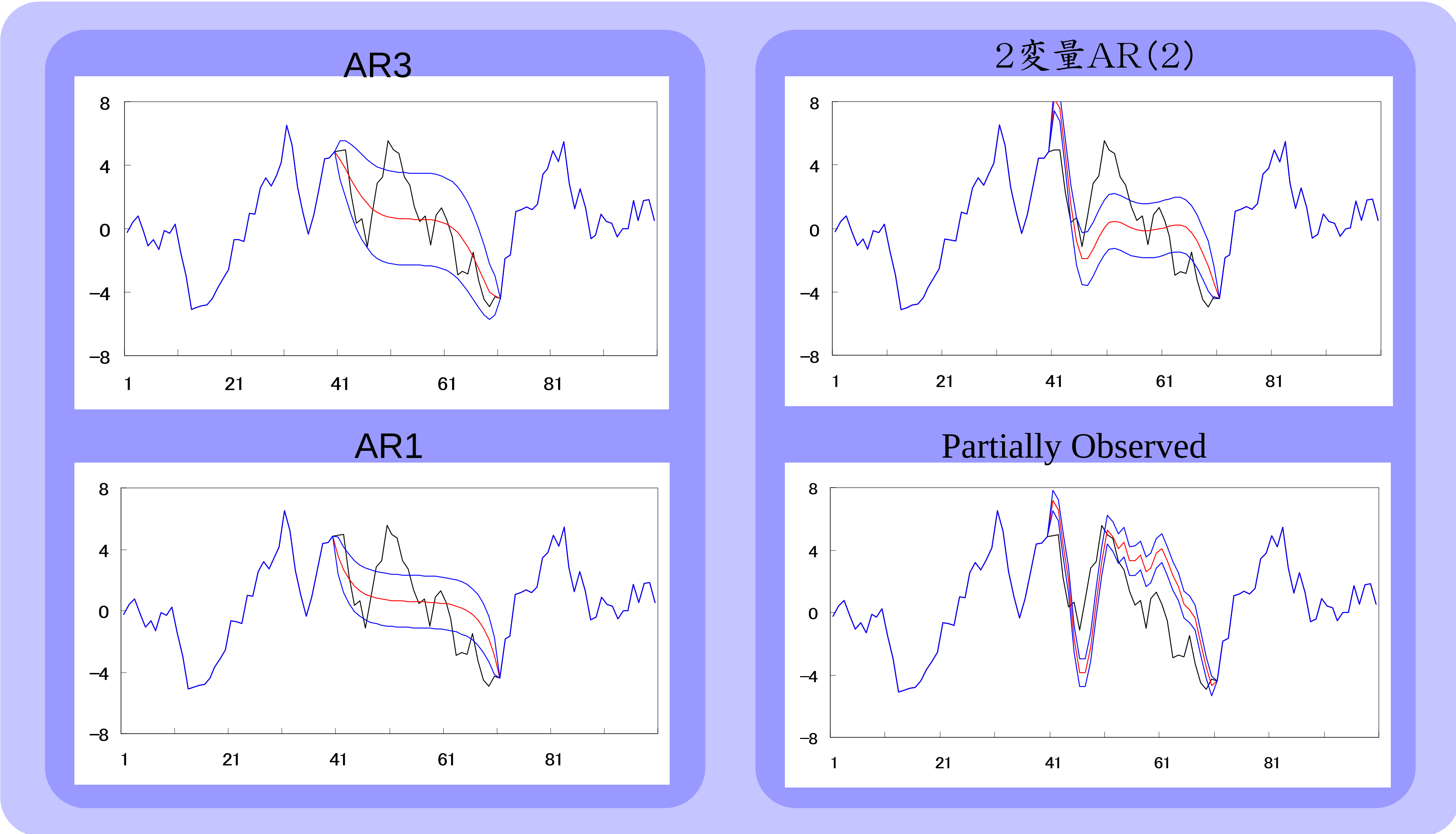
AR10



AR13

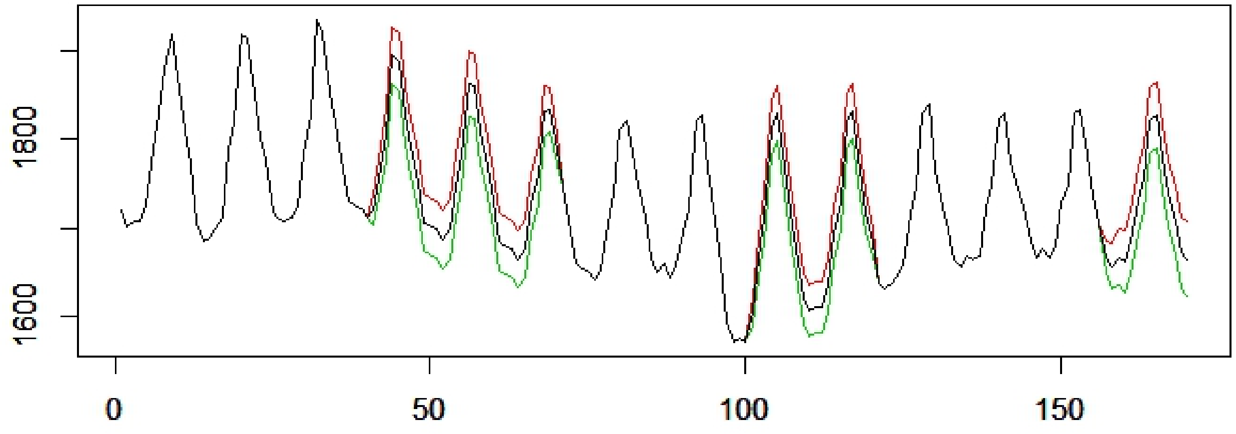


１変量補間と２変量補間

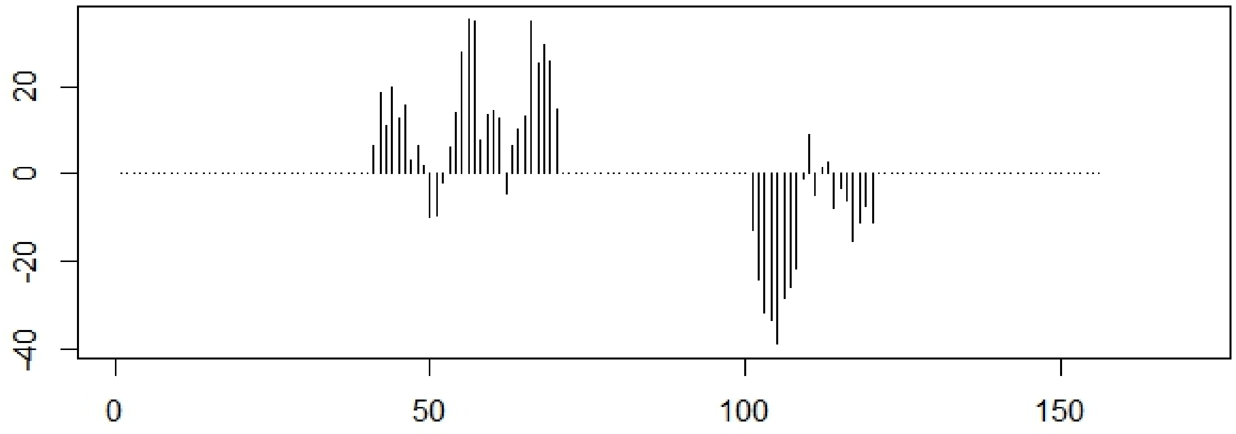


Rによる欠測値の補間（次数：自動）

```
## AR model (l=1, k=1)
data(Blsallfood) z1 <- arfit(Blsallfood, plot=FALSE)
m <- z1$maice.order
tau2 <- z1$sigma2[m+1]
arcoef <- z1$arcoef

f <- matrix(0.0e0, m, m)
f[1,] <- arcoef
for( i in 2:m ) f[i,i-1] <- 1
g <- c(1, rep(0.0e0, m-1))
h <- c(1, rep(0.0e0, m-1))
q <- tau2 r <- 0.0e0
x0 <- rep(0.0e0, m)
v0 <- NULL
z <- tsmooth(Blsallfood, f, g, h, q, r, x0, v0, 156, 170,
missed=c(41,101), np=c(30,20))
# plot mean vector and estimation error
xss <- z$mean.smooth[1,] + mean(Blsallfood)
cov <- z$cov.smooth
c1 <- xss + sqrt(cov[1,])
c2 <- xss - sqrt(cov[1,])
err <- z$esterr
par(mfcol=c(2,1))
ymax <- as.integer(max(xss,c1,c2)+1)
ymin <- as.integer(min(xss,c1,c2)-1)
plot(c1, type=‘l’, ylim=c(ymin,ymax), col=2, xlab=“Mean
vectors of the smoother XSS(1,) +/- standard deviation",
ylab="")
par(new=TRUE)
plot(c2, type='l', ylim=c(ymin,ymax), col=3, xlab="",
ylab="")
par(new=TRUE)
plot(xss, type='l', ylim=c(ymin,ymax), xlab="", ylab="")
plot(err[,1,1], type='h', xlim=c(1,length(xss)),
xlab="estimation error", ylab="")
```



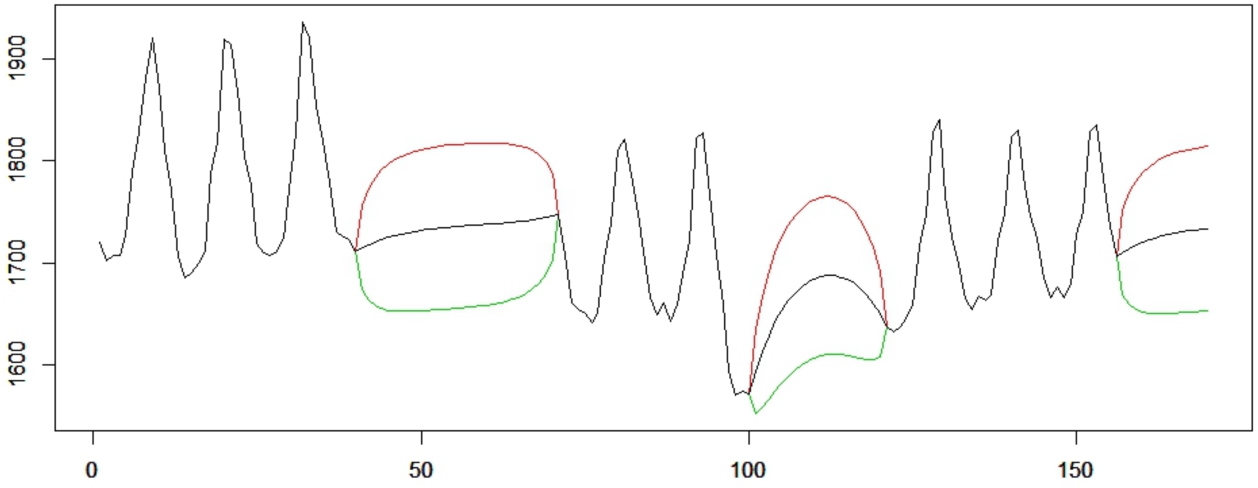
Mean vectors of the smoother XSS(1,) +/- standard deviation



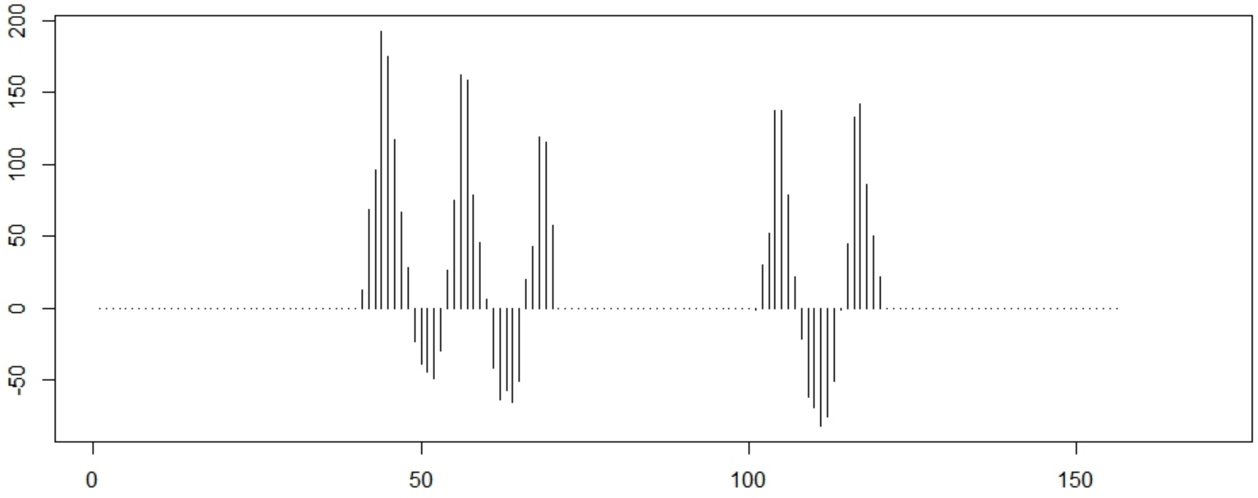
estimation error

Rによる欠測値の補間（AR次数＝１）

```
# AR order=1
  data(Blsallfood)
  z1 <- arfit(Blsallfood, plot=FALSE,lag=1)
  m <- z1$maice.order
  tau2 <- z1$sigma2[m+1]
  arcoef <- z1$arcoef
```



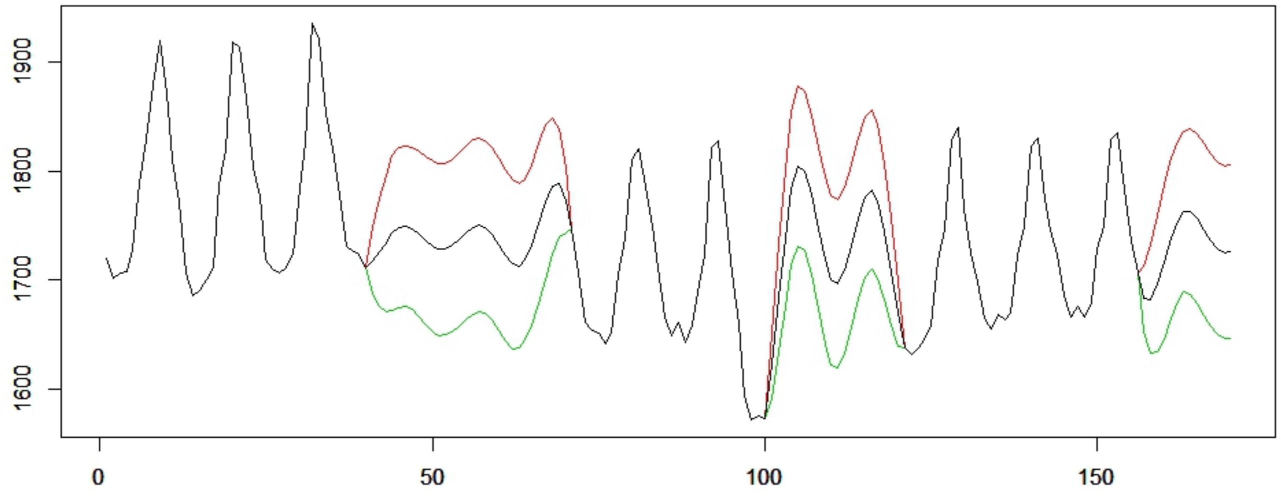
Mean vectors of the smoother XSS(1,) +/- standard deviation



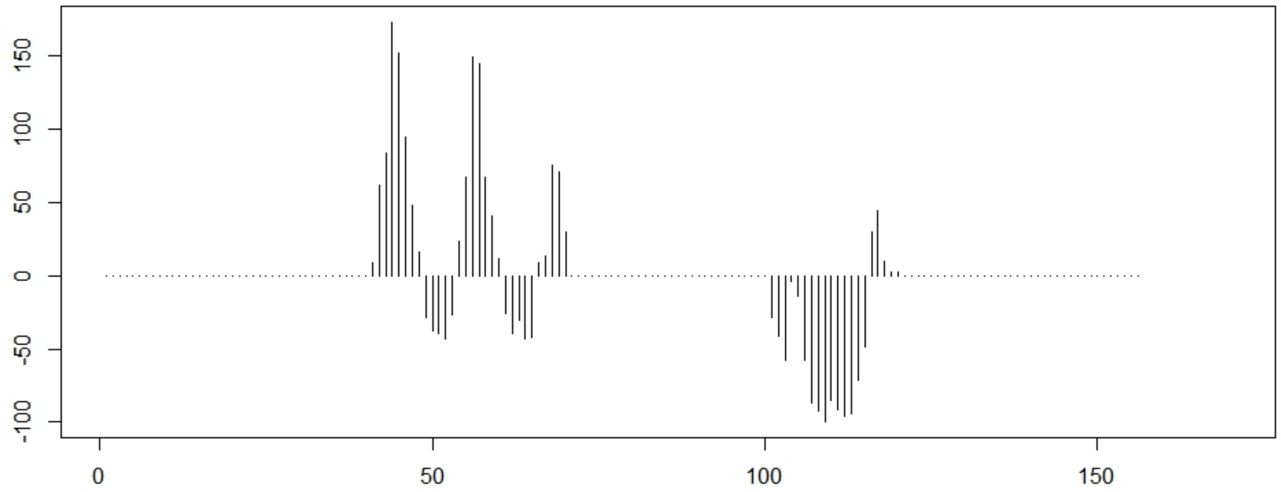
estimation error

Rによる欠測値の補間（AR次数＝5）

```
# AR order=5
  data(Blsallfood)
  z1 <- arfit(Blsallfood, plot=FALSE,lag=5)
  m <- z1$maice.order
  tau2 <- z1$sigma2[m+1]
  arcoef <- z1$arcoef
```



Mean vectors of the smoother XSS(1,) +/- standard deviation



estimation error

付録

カルマンフィルタの導出

予測ステップ

予測

$$\begin{aligned}
x_n &= F_n x_{n-1} + G_n v_n \\
x_{n|n-1} &= E\left[x_n \mid Y_{n-1}\right] \\
&= E\left[F x_{n-1} + G_n v_n \mid Y_{n-1}\right] \\
&= FE\left[x_{n-1} \mid Y_{n-1}\right] \\
&= F x_{n-1|n-1} \\
V_{n|n-1} &= E\left[(x_n - x_{n|n-1})^2\right] \\
&= E\left[(F(x_n - x_{n|n-1}) + G v_n)^2\right] \\
&= FE\left[(x_{n-1} - x_{n-1|n-1})^2\right] F^T + GE\left[v_n^2\right] G^T \\
&= F V_{n-1|n-1} F^T + GQG^T
\end{aligned}$$

フィルタステップ

フィルタ

$$
\begin{aligned}
\varepsilon_n &\equiv y_n - \mathrm{E}\left[y_n \mid Y_{n-1}\right] \\
&= Hx_n + w_n - \mathrm{E}\left[Hx_n + w_n \mid Y_{n-1}\right] \\
&= Hx_n + w_n - H\mathrm{E}\left[x_n \mid Y_{n-1}\right] \\
&= H(x_n - x_{n|n-1}) + w_n \\
\mathrm{Var}(\varepsilon_n) &= HV_{n|n-1}H^T + R \\
\mathrm{Cov}(x_n, \varepsilon_n) &= \mathrm{Cov}(x_n, H(x_n - x_{n|n-1}) + w_n) \\
&= \mathrm{Var}(x_n - x_{n|n-1})H_n^T \\
&= V_{n|n-1}H_n^T
\end{aligned}
$$

$$
Y_n = \left\{Y_{n-1}, y_n\right\} = Y_{n-1} \oplus \varepsilon_n
$$

$$
\begin{aligned}
x_{n|n} &= \mathrm{E}\left[x_n \mid Y_n\right] = \mathrm{Proj}\left[x_n \mid Y_{n-1}\right] \\
&= \mathrm{Proj}\left[x_n \mid Y_{n-1}, \varepsilon_n\right] \\
&= \mathrm{Proj}\left[x_n \mid Y_{n-1}\right] + \mathrm{Proj}\left[x_n \mid \varepsilon_n\right]
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Proj}\left[x_n \mid \varepsilon_n\right] &= \mathrm{Cov}(x_n, \varepsilon_n)\mathrm{Var}(\varepsilon_n)^{-1}\varepsilon_n \\
&= V_{n|n-1}H^T(HV_{n|n-1}H^T + R)^{-1}\varepsilon_n \\
&\equiv K_n\varepsilon_n \\
x_{n|n} &= x_{n|n-1} + K_n\varepsilon_n \\
V_{n|n-1} &= E\left[(x_n - x_{n|n-1})^2\right] \\
&= E\left[(x_n - x_{n|n} + K_n\varepsilon_n)^2\right] \\
&= V_{n|n} + K_n\mathrm{Var}(\varepsilon_n)K_n^T \\
V_{n|n} &= V_{n|n-1} - K_nHV_{n|n-1} \\
&= (I - K_nH)V_{n|n-1}
\end{aligned}
$$