時系列解析（7）

－多変量ARモデルの推定－
－局所定常ARモデル－
配布用

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Householder 法のメリット

Wは対角行列 ⇨ 行ごとに独立に推定できる

- モデリングが自由になる。変数ごと（説明変数、目的変数）に異なる次数を決められる
- 計算量が減少する　$(mk^2)^3 = m^3k^6$ ⇨ $k(mk)^3 = m^3k^4$　$\dfrac{m^3k^4}{m^3k^6} = \dfrac{1}{k^2}$

$$
X = [Z \mid y] = \overbrace{\left[\begin{array}{ccc|c}
y_m^T & \cdots & y_1^T & y_{m+1}^T \\
y_{m+1}^T & \cdots & y_2^T & y_{m+2}^T \\
\vdots & & \vdots & \vdots \\
\vdots & \ddots & \vdots & \vdots \\
y_{N-1}^T & \cdots & y_{N-m}^T & y_N^T
\end{array}\right]}^{k(m+1)} , \; N\text{-}m
\quad \Rightarrow \quad
HX = \begin{bmatrix} S \\ O \end{bmatrix} =
\begin{array}{c}
\begin{array}{cc} \overleftrightarrow{\quad km \quad} & \overleftrightarrow{\quad k \quad} \end{array} \\
\left[\begin{array}{ccc|cc}
s_{11} & \cdots & s_{1,km+k} & \cdots & s_{1,km+k} \\
 & \ddots & \vdots & & \vdots \\
 & & s_{1,km+k} & \cdots & s_{1,km+k} \\
 & & & \ddots & \vdots \\
 & & & & s_{1,km+k} \\
 & & O & &
\end{array}\right]
\end{array}
$$

第1成分のモデル推定

$$S=\left[\begin{array}{ccc|c} s_{11} & \cdots & s_{1,km} & s_{1,km+1} \\ & \ddots & \vdots & \vdots \\ & & s_{km,km} & s_{km,km+1} \\ & & & s_{km+1,km+1} \end{array}\right] \quad km+1$$

$$y_n(1)=\sum_{i=1}^{j} b_i(1,1)\,y_{n-i}(1)+\cdots+\sum_{i=1}^{j} b_i(1,k)\,y_{n-i}(k)+w_n$$

$$\begin{array}{c} \overleftrightarrow{\quad km+1 \quad} \\ \overleftrightarrow{\ kj\ } \end{array}$$

$$S=\begin{bmatrix} s_{11} & \cdots & s_{1,kj} & \cdots & s_{1,km} & s_{1,km+1} \\ & \ddots & \vdots & & \vdots & \vdots \\ & & s_{kj,kj} & \cdots & s_{kj,km} & s_{kj,km+1} \\ & & & \ddots & \vdots & \vdots \\ & & & & s_{km,km} & s_{km,km+1} \\ & & & & & s_{km+1,km+1} \end{bmatrix}$$

$$\hat{\sigma}_j^2(1)=\frac{1}{N-m}\sum_{i=kj+1}^{km+1} s_{i,km+1}^2$$

$$\mathrm{AIC}_j(1)=(N-m)\log(2\pi\hat{\sigma}_j^2(1)+1)+2(kj+1)$$

$$\begin{bmatrix} s_{11} & \cdots & s_{1,kj} \\ & \ddots & \vdots \\ & & s_{kj,kj} \end{bmatrix}\begin{bmatrix} c_1 \\ \vdots \\ c_{kj} \end{bmatrix}=\begin{bmatrix} s_{1,km+1} \\ \vdots \\ s_{kj,km+1} \end{bmatrix}$$

第２成分のモデル推定

$$
S=\begin{bmatrix}
s_{11} & \cdots & s_{1,km} & s_{1,km+1} & s_{1,km+2} & \cdots & s_{1,km+k} \\
s_{21} & \cdots & s_{2,km} & & s_{2,km+2} & \cdots & s_{2,km+k} \\
 & \ddots & \vdots & & \vdots & & \vdots \\
 & & s_{km+1,km} & & s_{km+1,km+2} & \cdots & s_{km+1,km+k} \\
 & & & & s_{km+2,km+2} & \cdots & s_{km+1,km+k} \\
 & & & & & \ddots & \vdots \\
 & & & & & & s_{km+k,km+k}
\end{bmatrix}
$$

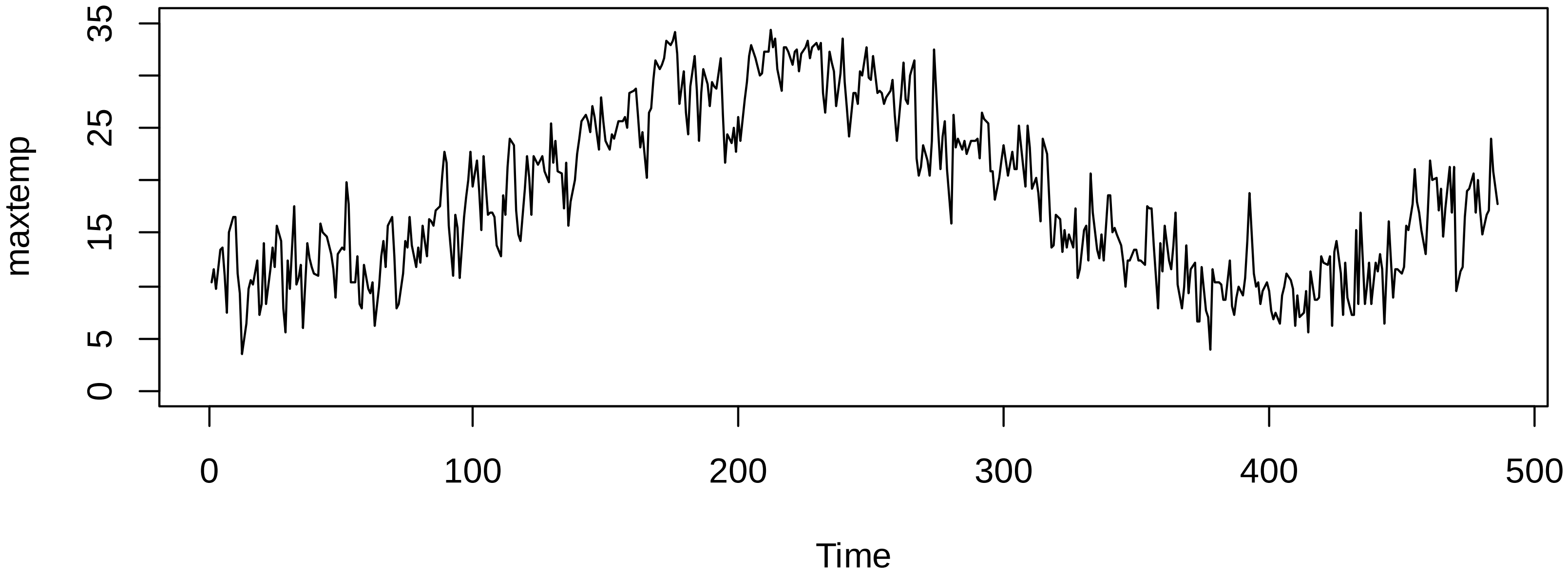
$$
y_n(2)=\sum_{i=1}^{j} b_i(2,1)y_{n-i}(1)+\cdots+\sum_{i=1}^{j} b_i(2,k)y_{n-i}(k)+w_n
$$

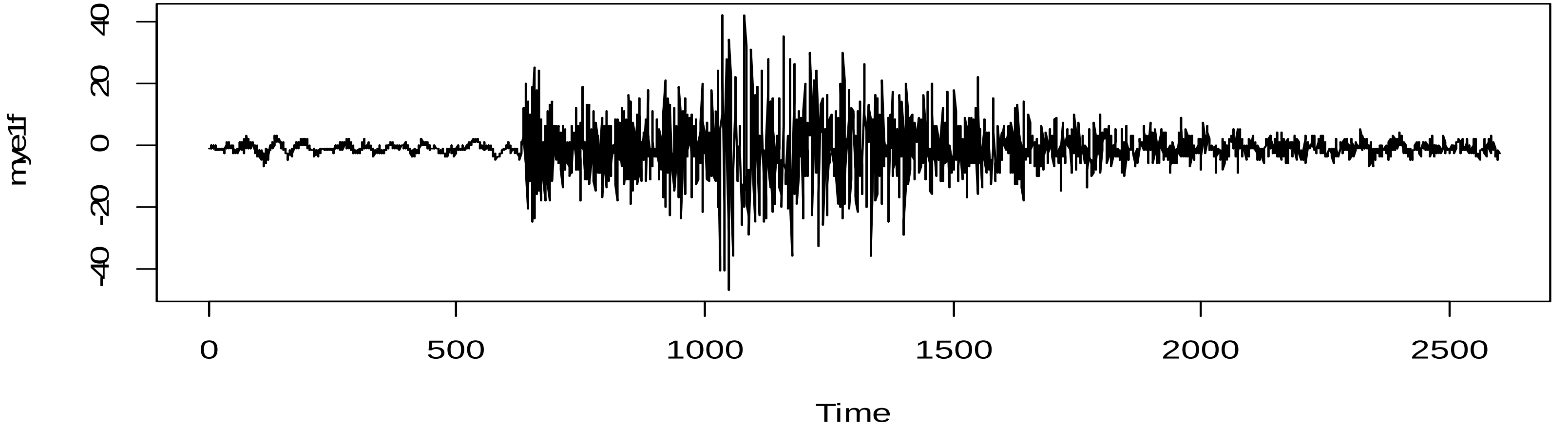
$$
\hat{\sigma}_j^2(2)=\frac{1}{N-m}\sum_{i=kj+2}^{km+2} s_{i,km+2}^2
$$

$$
\mathrm{AIC}_j(2)=(N-m)\log(2\pi\hat{\sigma}_j^2(2)+1)+2(kj+2)
$$

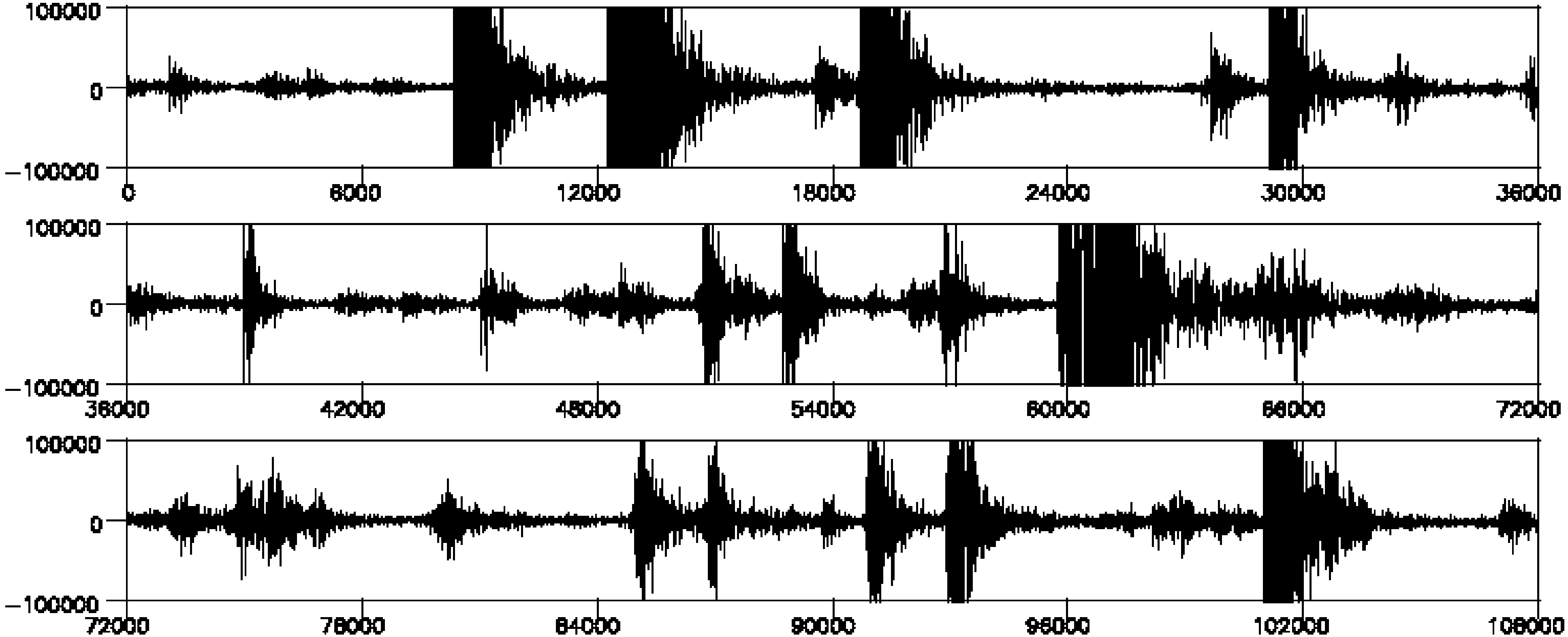
$$
\begin{bmatrix}
s_{11} & \cdots & s_{1,kj} & s_{1,km+1} \\
s_{21} & \cdots & s_{2,kj} & \\
 & \ddots & \vdots & \\
 & & s_{kj+1,kj} &
\end{bmatrix}
\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{kj+1} \end{bmatrix}
=
\begin{bmatrix} s_{1,km+2} \\ s_{2,km+2} \\ \vdots \\ s_{kj+1,km+2} \end{bmatrix}
$$

定常モデルで非定常時系列を分析する



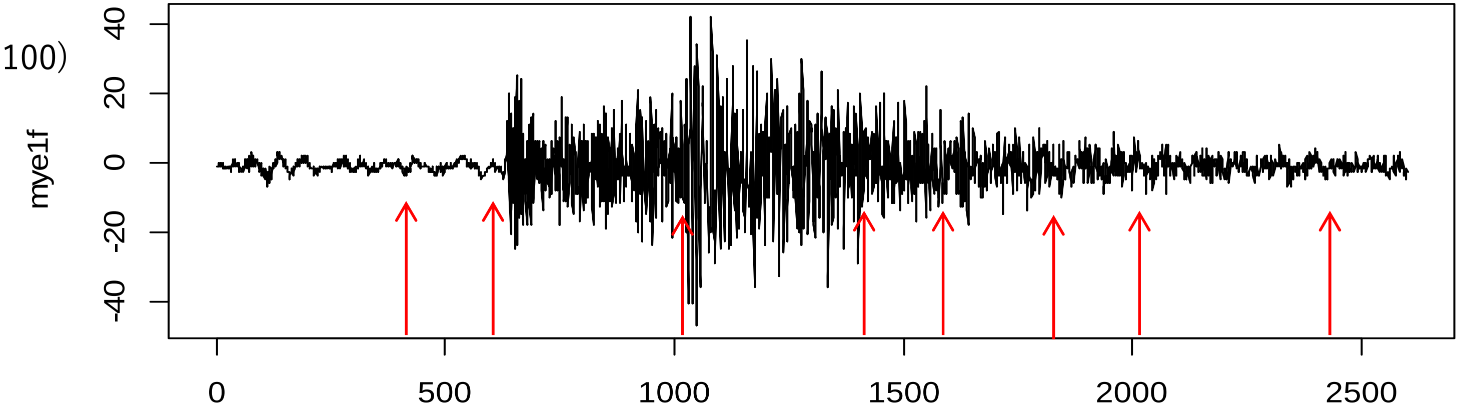


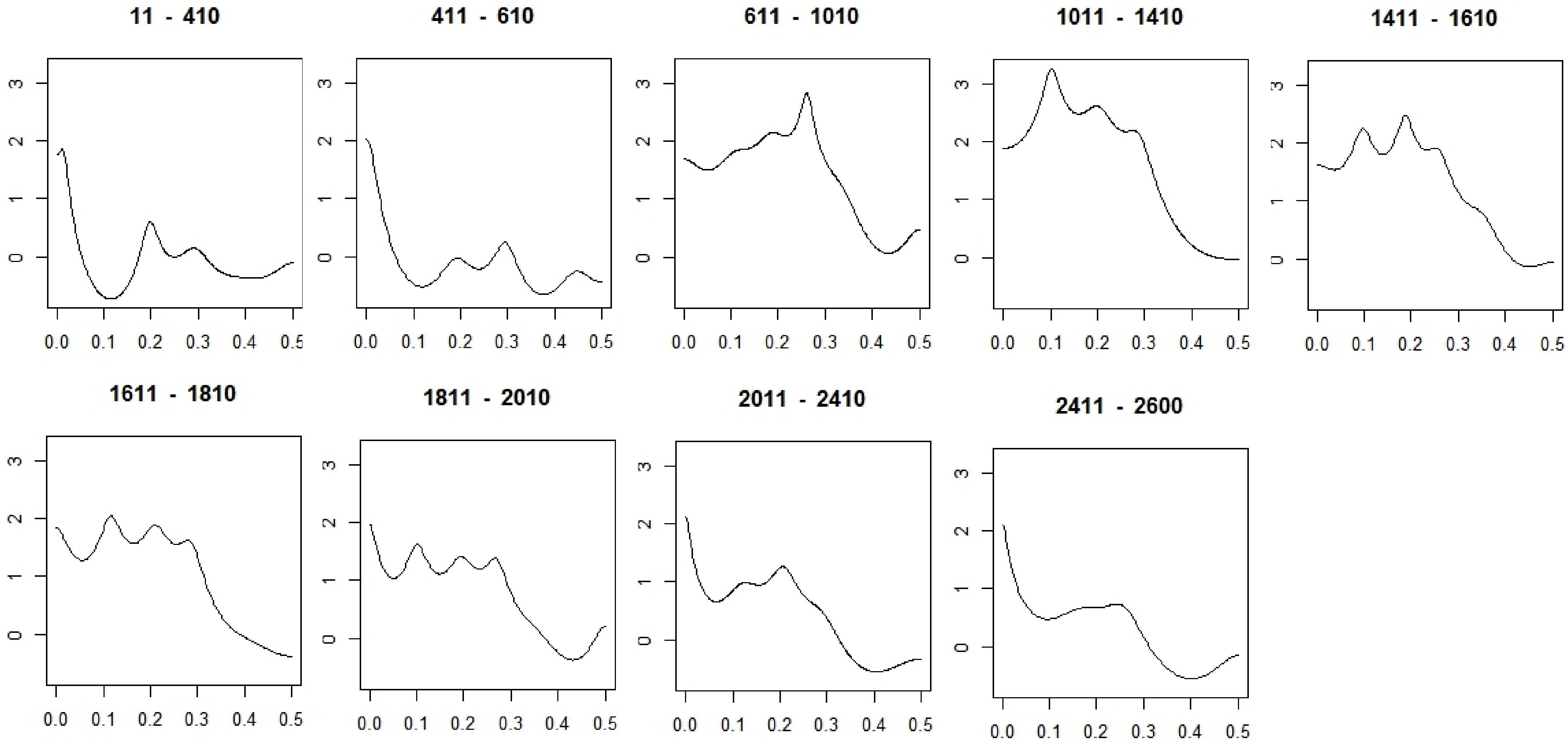
火山性微動（有珠山）



Hokkaido, Japan March 31, 2000 13:07-
北海道大学火山地震予知観測センター　高波鐵夫

data(MYE1F)
lsar1(MYE1F, 10, 100)

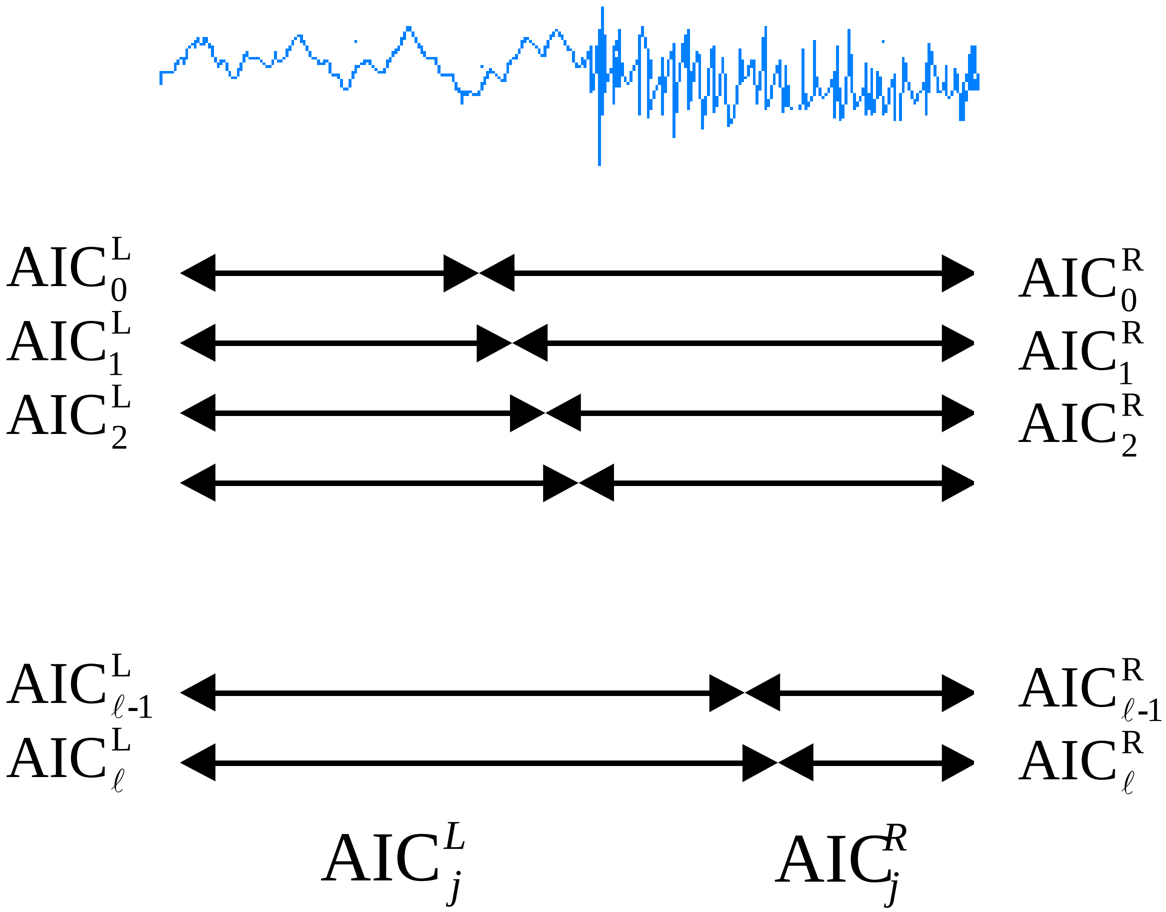




Locally Stationary AR Modeling

```
<<< new data ( n = 11 --- 210 ) >>>
    initial model : NS =   200
        ms = 9   sds =  9.533019e-01   aics =      578.011

<<< new data ( n = 211 --- 410 ) >>>
    switched model : ( nf = 200,  ns = 200 )
        ms = 8   sds =  7.571863e-01   aics =     1107.957
    pooled model : ( np = 400 )
        mp = 9   sdp =  8.775778e-01   aicp =     1102.915
 ***  pooled model accepted   ***

<<< new data ( n = 411 --- 610 ) >>>
    switched model : ( nf = 400,  ns = 200 )
        ms = 8   sds =  7.353224e-01   aics =     1627.001
    pooled model : ( np = 600 )
        mp = 9   sdp =  8.567912e-01   aicp =     1629.990
 ***  switched model accepted   ***

<<< new data ( n = 611 --- 810 ) >>>
    switched model : ( nf = 200,  ns = 200 )
        ms = 4   sds =  2.372409e+01   aics =     1734.960
    pooled model : ( np = 400 )
        mp = 10   sdp =  1.255320e+01   aicp =     2169.141
 ***  switched model accepted   ***

<<< new data ( n = 811 --- 1010 ) >>>
    switched model : ( nf = 200,  ns = 200 )
        ms = 4   sds =  2.701266e+01   aics =     2447.710
    pooled model : ( np = 400 )
        mp = 10   sdp =  2.468140e+01   aicp =     2439.571
 ***  pooled model accepted   ***

<<< new data ( n = 1011 --- 1210 ) >>>
    switched model : ( nf = 400,  ns = 200 )
        ms = 9   sds =  4.807289e+01   aics =     3801.690
    pooled model : ( np = 600 )
        mp = 10   sdp =  3.864933e+01   aicp =     3917.444

 ***  switched model accepted   ***

<<< new data ( n = 1211 --- 1410 ) >>>
    switched model : ( nf = 200,  ns = 200 )
        ms = 9   sds =  3.470873e+01   aics =     2659.093
    pooled model : ( np = 400 )
        mp = 8   sdp =  4.308581e+01   aicp =     2658.428

 ***  pooled model accepted   ***

<<< new data ( n = 1411 --- 1610 ) >>>
    switched model : ( nf = 400,  ns = 200 )
        ms = 10   sds =  1.764112e+01   aics =     3822.050
    pooled model : ( np = 600 )
        mp = 8   sdp =  3.582855e+01   aicp =     3867.973

 ***  switched model accepted   ***

<<< new data ( n = 1611 --- 1810 ) >>>
    switched model : ( nf = 200,  ns = 200 )
        ms = 9   sds =  1.032457e+01   aics =     2218.103
    pooled model : ( np = 400 )
        mp = 10   sdp =  1.447380e+01   aicp =     2226.087

 ***  switched model accepted   ***
```

変化時点の精密な推定

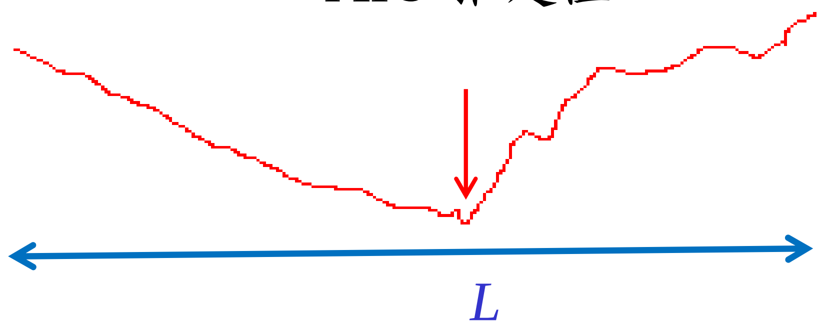


全体のモデルのAIC

$$\mathrm{AIC}_j = \mathrm{AIC}_j^L + \mathrm{AIC}_j^R$$

変化時点の最小

AIC 推定値



データの逐次追加

$2m$個のARモデルをL回推定 ⇒ 自動処理のための高速化

$2mL \quad (m=10,\ L=100\sim500)$ 　　　　 $6m$回程度

$$X_0 = [Z\,|\,y] = \begin{bmatrix} y_m & \cdots & y_1 & y_{m+1} \\ y_{m+1} & \cdots & y_2 & y_{m+2} \\ \vdots & \ddots & \vdots & \vdots \\ y_{n_0-1} & \cdots & y_{n_0-m} & y_{n_0} \end{bmatrix} \Rightarrow HX = \begin{bmatrix} S \\ \\ O \end{bmatrix} = \begin{bmatrix} s_{11} & \cdots & s_{1,m+1} \\ & \ddots & \vdots \\ & & s_{m+1,m+1} \\ & O & \end{bmatrix}$$

$$\hat{\sigma}_0^2(j) = \frac{1}{n_0 - m} \sum_{i=j+1}^{m+1} s_{i,m+1}^2$$

$$\mathrm{AIC}_0(j) = (n_0 - m)\log \hat{\sigma}_0^2(j) + 2(j+1)$$

$$\mathrm{AIC}_0^1 \equiv \min_j \mathrm{AIC}_0(j)$$

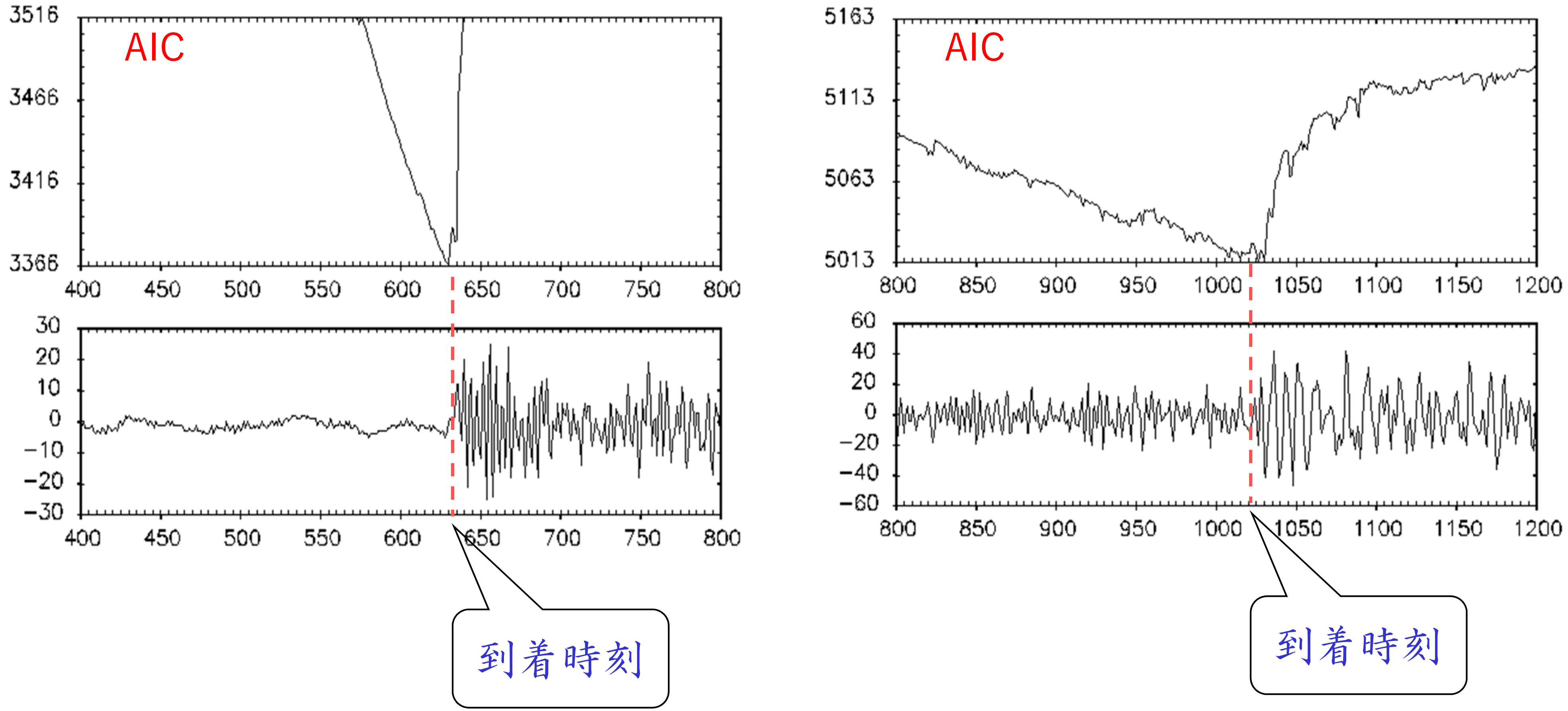
$$X_1 = \begin{bmatrix} s_{1,1} & \cdots & s_{1,m} & s_{1,m+1} \\ & \ddots & \vdots & \vdots \\ & & s_{m,m} & s_{m,m+1} \\ & & & s_{m+1,m+1} \\ \hline y_{n_0} & \cdots & y_{n_0-m+1} & y_{n_0+1} \\ \vdots & & \vdots & \vdots \\ y_{n_0+p-1} & \cdots & y_{n_0-m+p} & y_{n_0+p} \end{bmatrix} \Rightarrow H_1 X_1 = \begin{bmatrix} R \\ O \end{bmatrix} = \begin{bmatrix} s_{11} & \cdots & s_{1,m} & s_{1,m+1} \\ & \ddots & \vdots & \vdots \\ & & s_{m,m} & s_{m,m+1} \\ & & & s_{m+1,m+1} \\ \hline & & O & \end{bmatrix}$$

$$\hat{\sigma}_1^2(j) = \frac{1}{n_0 - m + p} \sum_{i=j+1}^{m+1} r_{i,m+1}^2$$

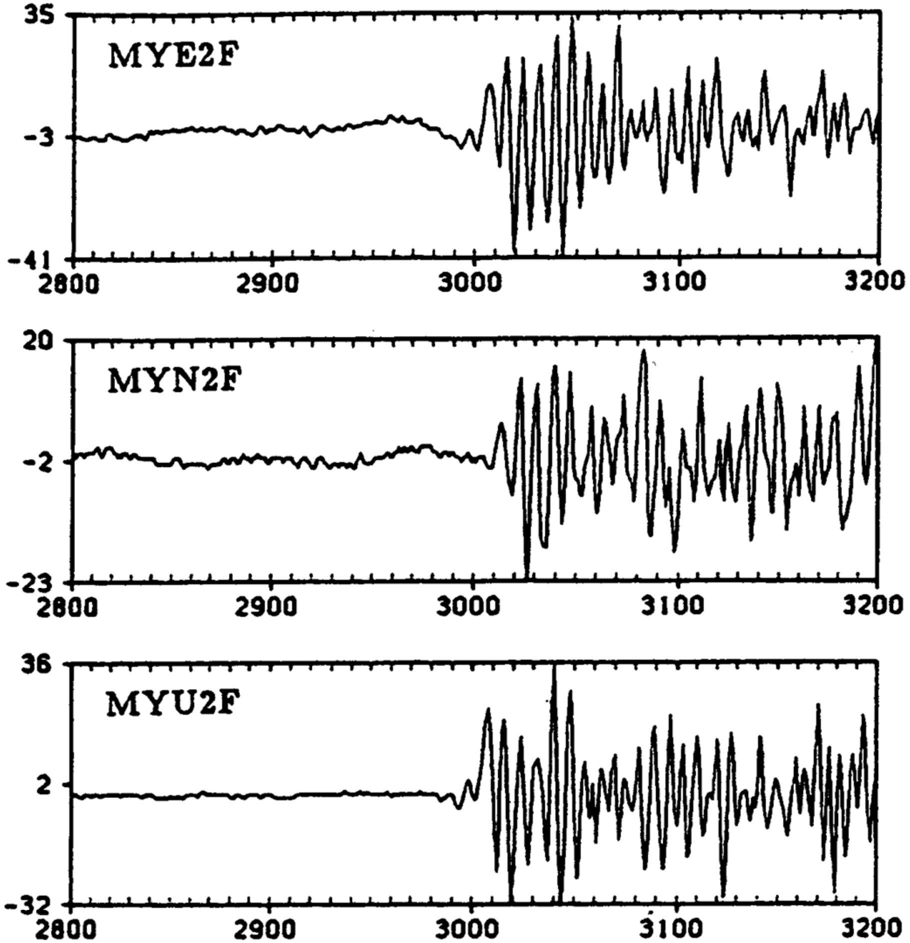
$$\mathrm{AIC}_1(j) = (n_0 - m + p)\log \hat{\sigma}_1^2(j) + 2(j+1)$$

$$\mathrm{AIC}_1^1 \equiv \min_j \mathrm{AIC}_1(j)$$

地震波到着時刻の推定



AIC
3516
3466
3416
3366
400 450 500 550 600 650 700 750 800
30
20
10
0
−10
−20
−30
400 450 500 550 600 650 700 750 800
到着時刻
AIC
5163
5113
5063
5013
800 850 900 950 1000 1050 1100 1150 1200
60
40
20
0
−20
−40
−60
800 850 900 950 1000 1050 1100 1150 1200
到着時刻


MYE2F
MYN2F
MYU2F

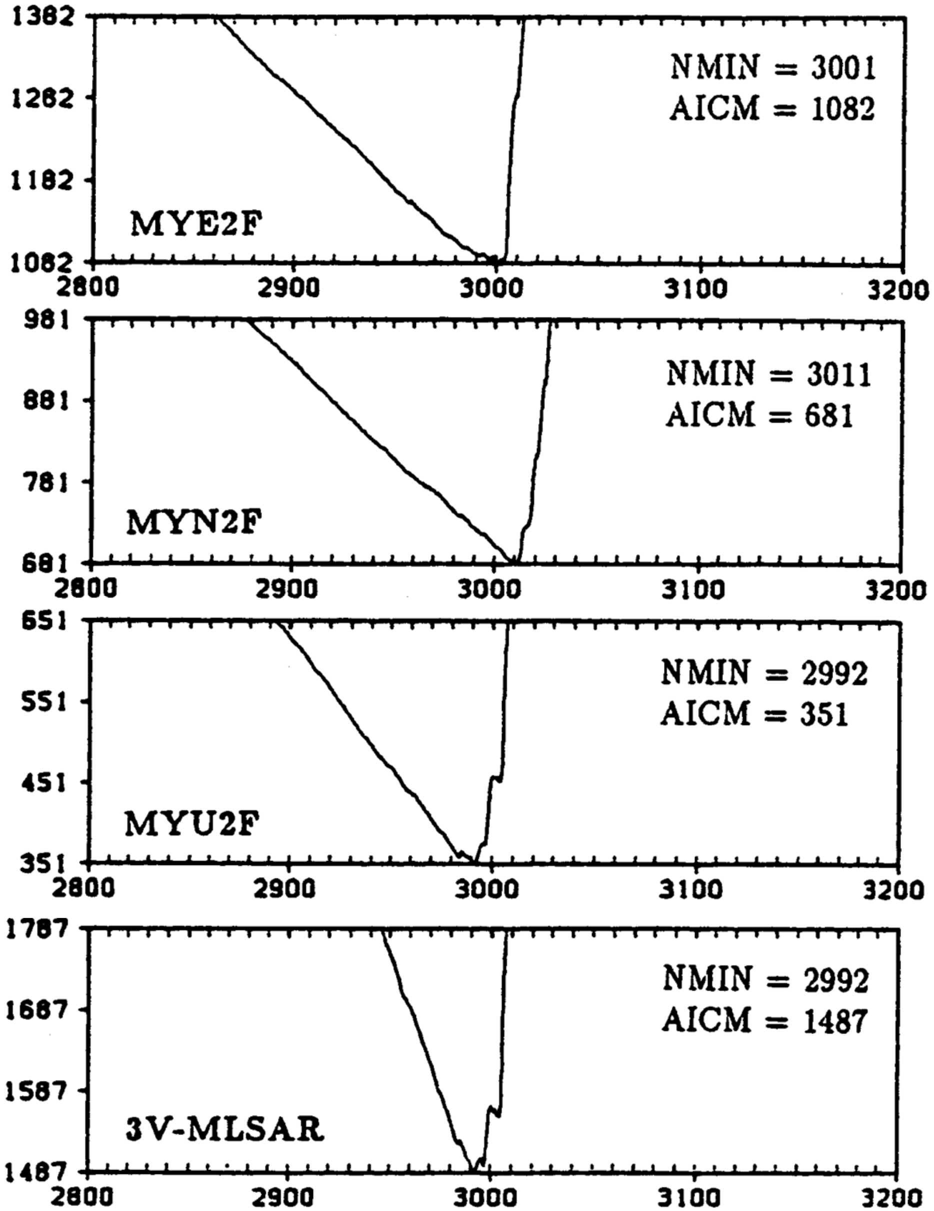

NMIN = 3001
AICM = 1082
MYE2F
NMIN = 3011
AICM = 681
MYN2F
NMIN = 2992
AICM = 351
MYU2F
NMIN = 2992
AICM = 1487
3V-MLSAR

Fig. 3.1

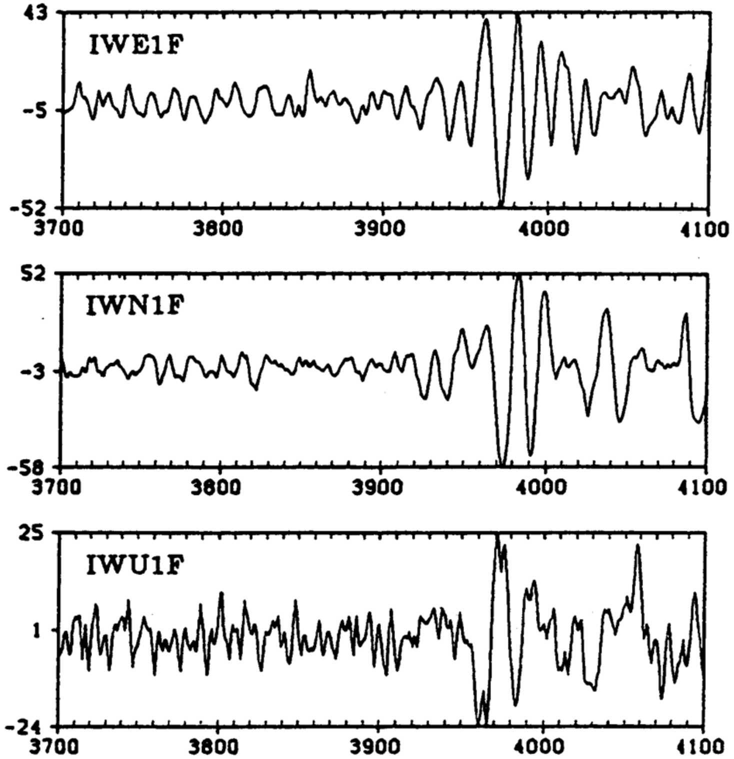
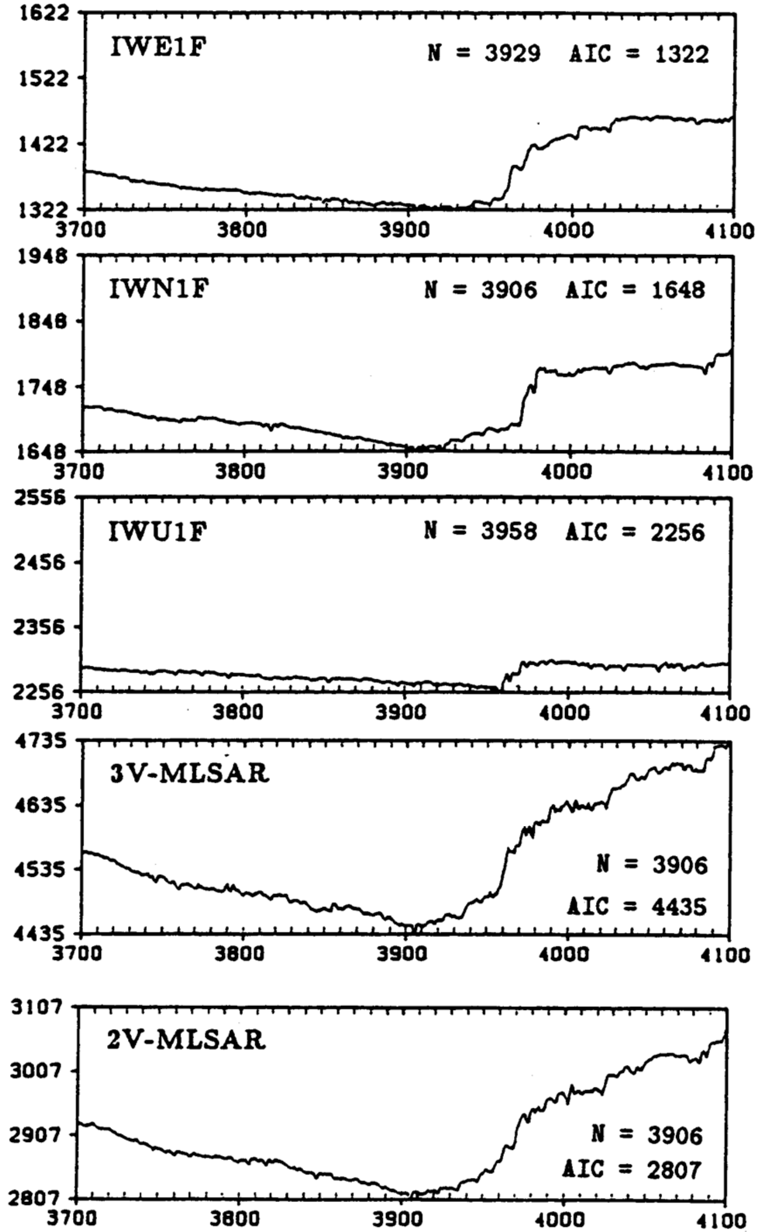

IWE1F
IWN1F
IWU1F

Fig. 5.1


IWE1F
N = 3929 AIC = 1322
IWN1F
N = 3906 AIC = 1648
IWU1F
N = 3958 AIC = 2256
3V-MLSAR
N = 3906
AIC = 4435
2V-MLSAR
N = 3906
AIC = 2807

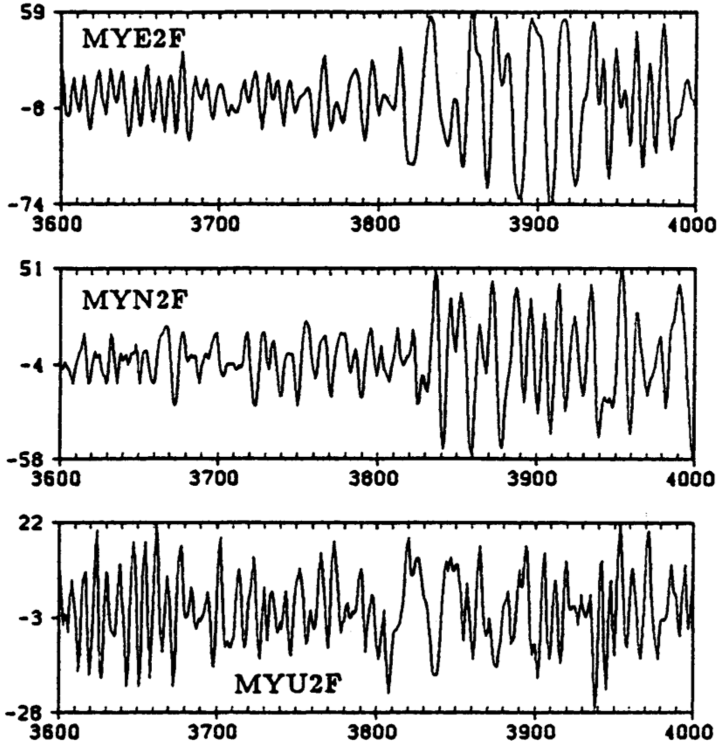
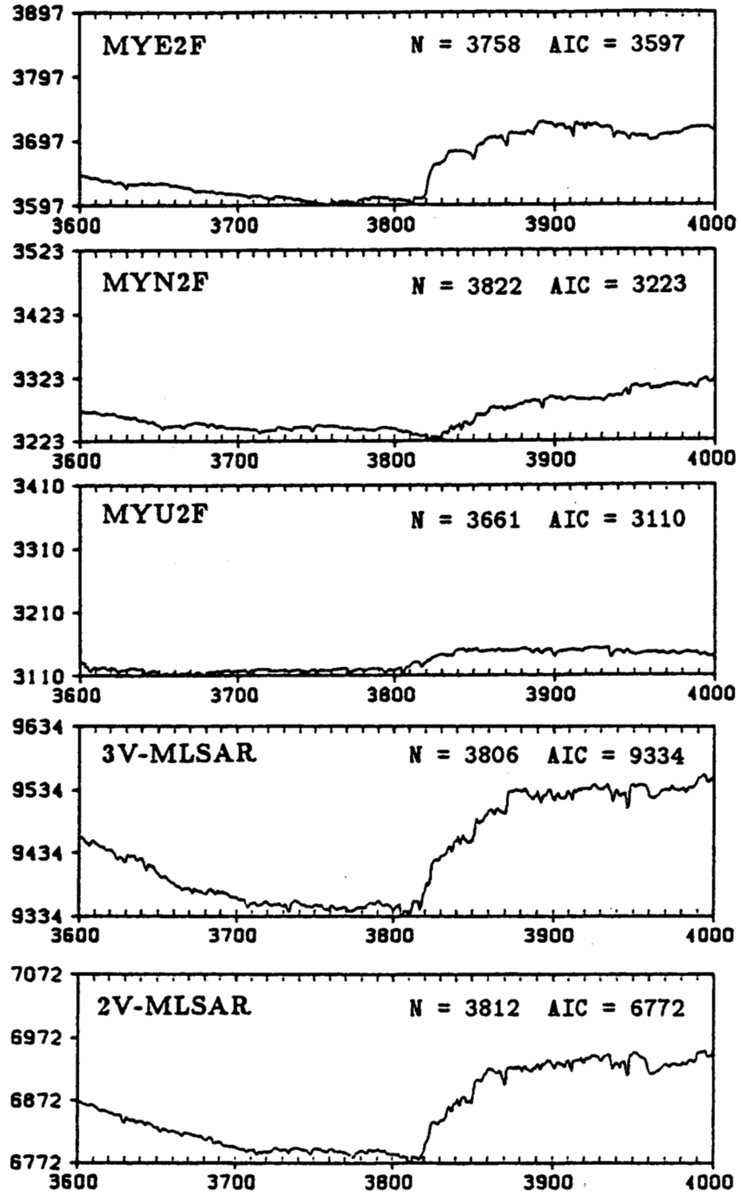
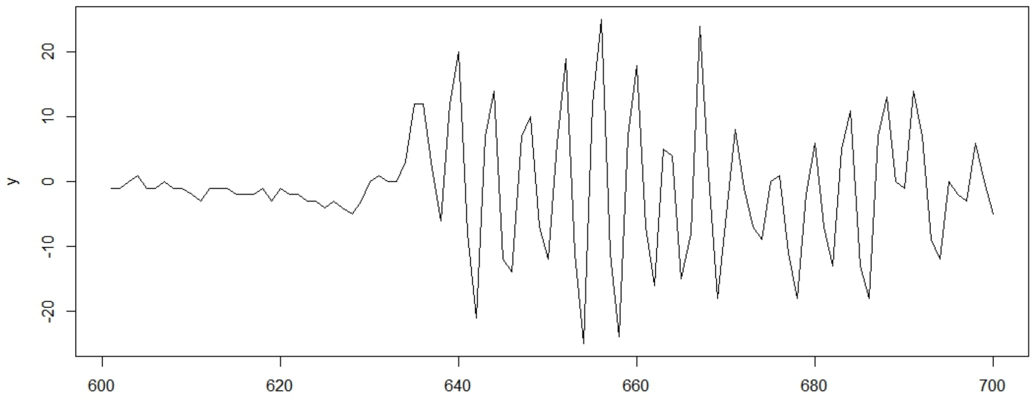

MYE2F
59
-8
-74
MYN2F
51
-4
-58
MYU2F
22
-3
-28
3600
3700
3800
3900
4000

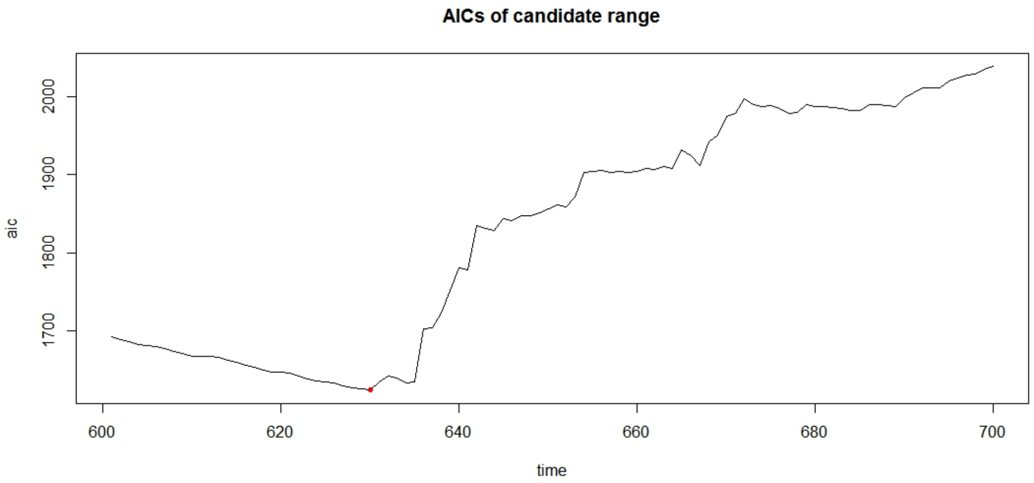
Fig. 7.1


MYE2F
N = 3758 AIC = 3597
3897
3797
3697
3597
MYN2F
N = 3822 AIC = 3223
3523
3423
3323
3223
MYU2F
N = 3661 AIC = 3110
3410
3310
3210
3110
3V-MLSAR
N = 3806 AIC = 9334
9634
9534
9434
9334
2V-MLSAR
N = 3812 AIC = 6772
7072
6972
6872
6772
3600
3700
3800
3900
4000

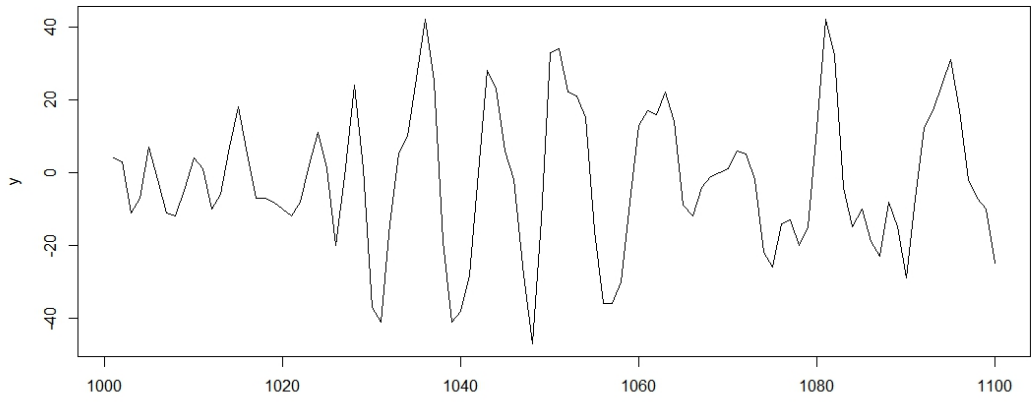
Rによる計算

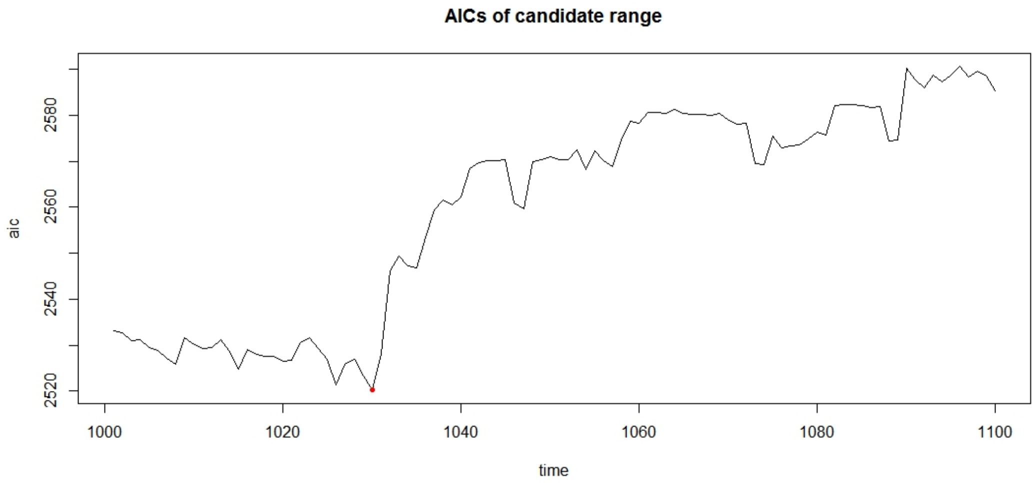
```
data(MYE1F)
lsar2(MYE1F, 10, c(400,800), c(600,700))
```






```
data(MYE1F)
lsar2(MYE1F, 10, c(800,1200), c(1000,1100))
```





変化点の事後確率

変化点の事後確率

AIC ＝ − 2 ×（バイアス補正した対数尤度）

$$p(y\,|\,j) \equiv \exp\left\{ -\frac{1}{2}\mathrm{AIC}_j \right\}$$ 最尤法でパラメータ推定したモデルの尤度

$p(j)$ 変化点の事前確率

$$p(j\,|\,y) = \frac{p(y\,|\,j)\,p(j)}{\sum_{j=1} p(y\,|\,j)\,p(j)}$$ 変化点の事後確率

```
x <- lsar2(MYE1F, 10, c(400,800), c(600,700))
AICP <- x$aic
post <- exp( -(AICP-min(AICP))/2 )
post <- post/sum(post)
plot( post, type="l", col="red“ )
```

```
x <- lsar2(MYE1F, 10, c(400,800), c(600,700))
AICP <- x$aic
post <- exp( -(AICP-min(AICP))/2 )
post <- post/sum(post)
plot(post,type="l",col="red",lwd=2)
```