時系列解析（6）

－ ARモデルの推定 －

－ 配布用 －

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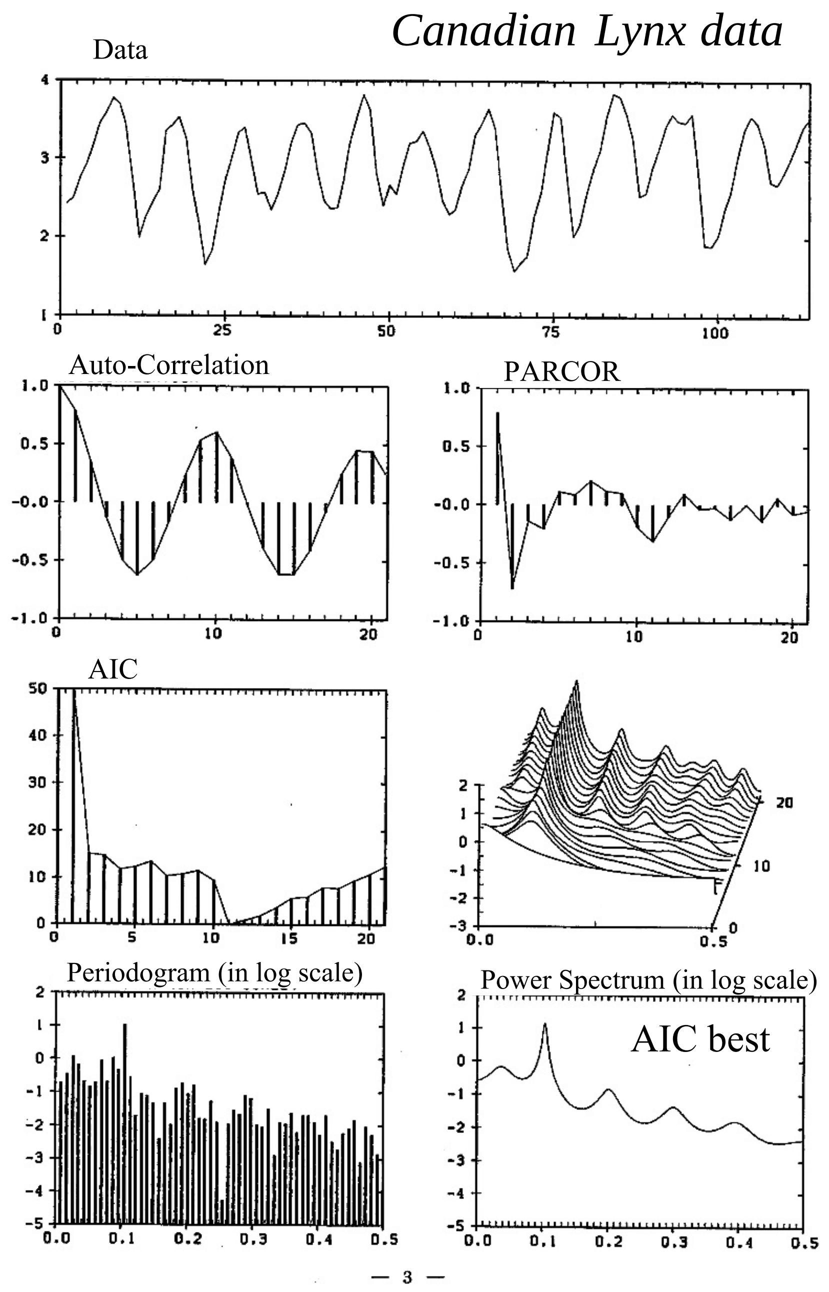
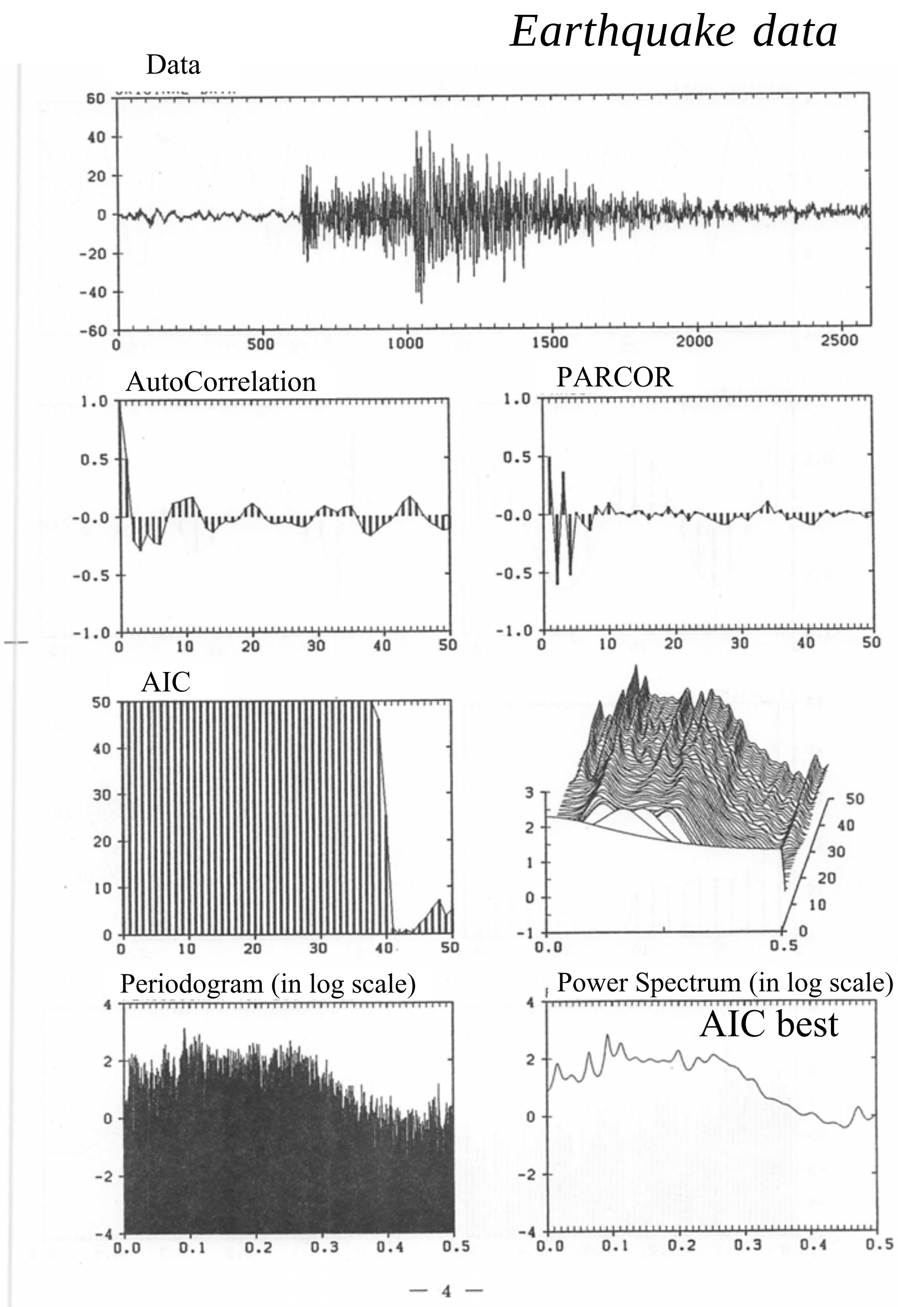
次数によって結果は変わる



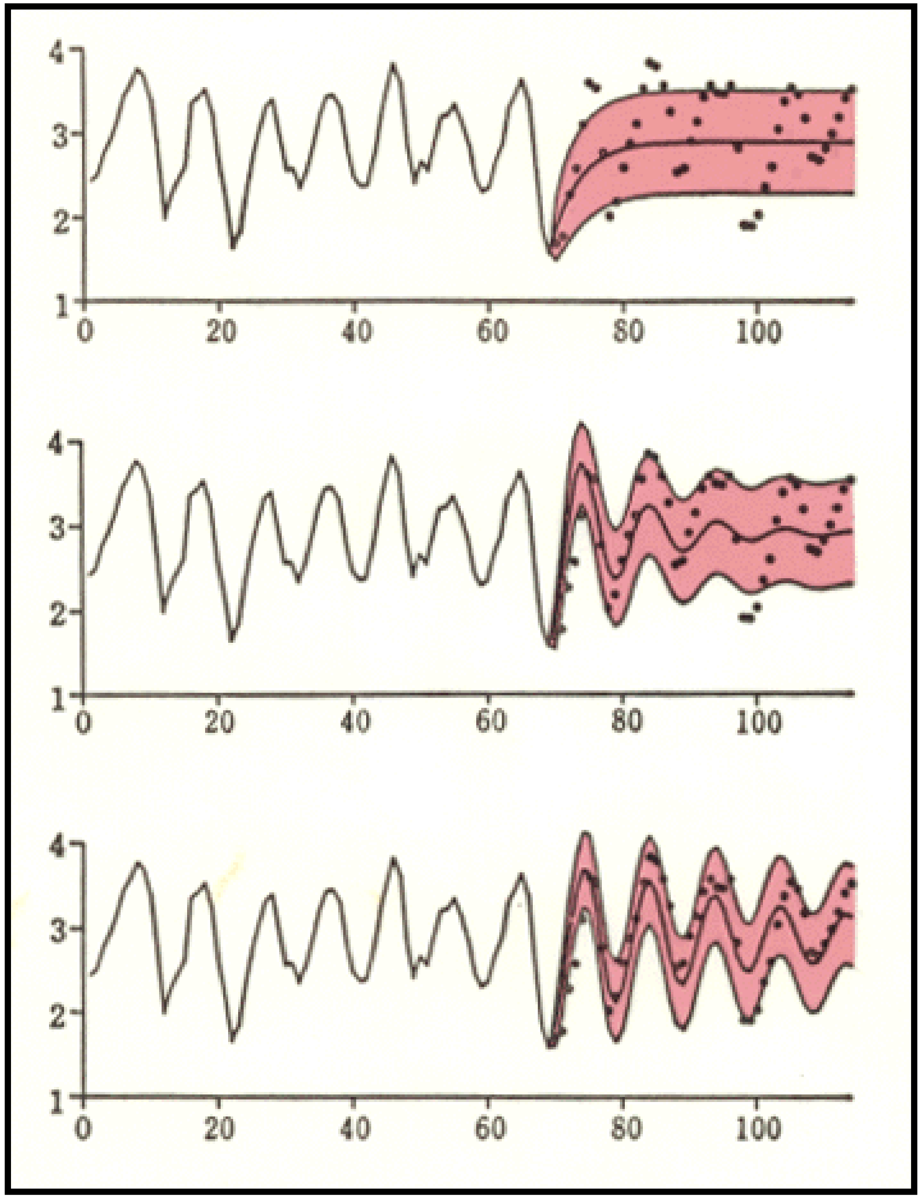


次数によって結果は変わる

長期予測

Canadian Lynx data

$$y_n = \sum_{j=1}^{m} a_j y_{n-j} + v_n$$

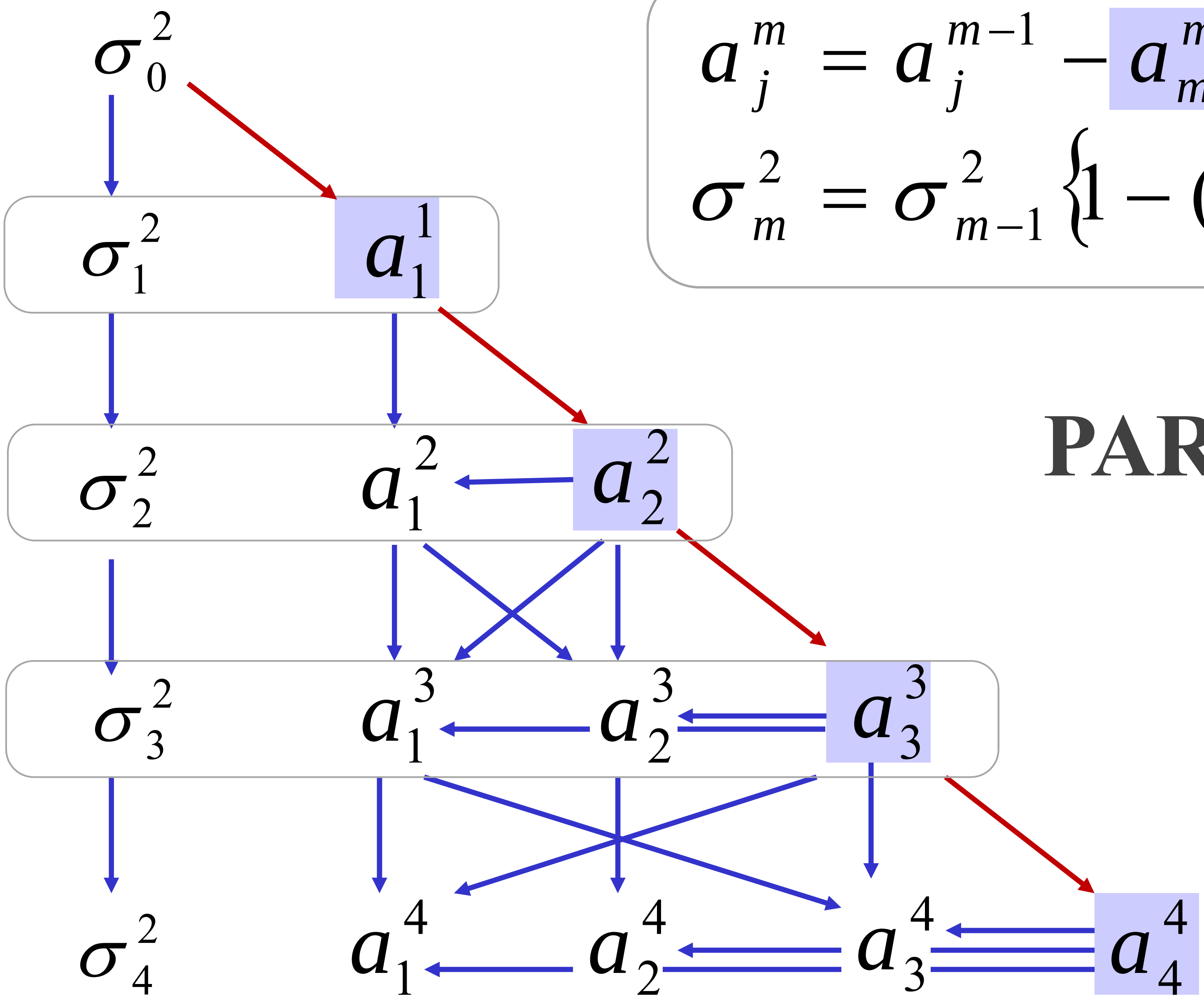


$m = 1$

$m = 5$

$m = 10$

Levinson's　Algorithm



$$a_j^m = a_j^{m-1} - \boxed{a_m^m}\, a_{m-j}^{m-1}$$

$$\sigma_m^2 = \sigma_{m-1}^2 \left\{ 1 - \left(\boxed{a_m^m} \right)^2 \right\}$$

PARCOR

Householder 法

$$Z = \begin{bmatrix} y_m & y_{m-1} & \cdots & y_1 \\ y_{m+1} & y_m & \cdots & y_2 \\ \vdots & \vdots & \ddots & \vdots \\ y_{N-1} & y_{N-2} & \cdots & y_{N-m} \end{bmatrix}, \quad y = \begin{bmatrix} y_{m+1} \\ y_{m+2} \\ \vdots \\ y_N \end{bmatrix}$$

$$Z = [X \mid y] = \left[\begin{array}{cccc|c} y_m & y_{m-1} & \cdots & y_1 & y_{m+1} \\ y_{m+1} & y_m & \cdots & y_2 & y_{m+2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ y_{N-1} & y_{N-2} & \cdots & y_{N-m} & y_N \end{array}\right]$$

$$HX = \begin{bmatrix} S \\ O \end{bmatrix} = \left[\begin{array}{ccc|c} s_{11} & \cdots & s_{1m} & s_{1,m+1} \\ & \ddots & \vdots & \vdots \\ & & s_{mm} & s_{m,m+1} \\ & & & s_{m+1,m+1} \\ \hline \multicolumn{4}{c}{O} \end{array}\right]$$

$$\hat{\sigma}_k^2 = \frac{1}{N-m} \sum_{i=k+1}^{m+1} s_{i,m+1}^2$$

$$\mathrm{AIC}_k = (N-m)(\log 2\pi\hat{\sigma}_k^2 + 1) + 2(k+1)$$

$$\begin{bmatrix} s_{11} & \cdots & s_{1k} \\ & \ddots & \vdots \\ & & s_{kk} \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_k \end{bmatrix} = \begin{bmatrix} s_{1,m+1} \\ \vdots \\ s_{k,m+1} \end{bmatrix}$$

AICによる次数選択

$$\ell(\hat{\theta}) = -\frac{N}{2}\log 2\pi\hat{\sigma}_m^2 - \frac{N}{2}$$

$$\begin{aligned}\mathrm{AIC}_m &= -2\ell(\hat{\theta}) + 2(\text{パラメータ数})\\ &= N(\log 2\pi\hat{\sigma}_m^2 + 1) + 2(m+1)\end{aligned}$$

$$\begin{bmatrix} s_{11} & \cdots & s_{1k} & \cdots & s_{1m} & s_{1,m+1} \\ & \ddots & \vdots & & \vdots & \vdots \\ & & s_{kk} & \cdots & s_{km} & s_{k,m+1} \\ & & & \ddots & \vdots & \vdots \\ & & & & s_{mm} & s_{m,m+1} \\ & & & & & s_{m+1,m+1} \\ & & 0 & & & \end{bmatrix}$$

for $k = 1,\ldots,m$

$$\hat{\sigma}_k^2 = \frac{1}{n}\left(s_{k+1,m+1}^2 + \cdots + s_{m+1,m+1}^2\right)$$

$$\mathrm{AIC}_k = N(\log 2\pi\hat{\sigma}_k^2 + 1) + 2(k+1)$$

以下の計算はしなくてもよい

$$\begin{bmatrix} s_{11} & \cdots & s_{1k} \\ & \ddots & \vdots \\ & & s_{kk} \end{bmatrix}\begin{bmatrix} a_1 \\ \vdots \\ a_k \end{bmatrix} = \begin{bmatrix} s_{1,m+1} \\ \vdots \\ s_{k,m+1} \end{bmatrix}$$

（４）PARCOR法によるARモデルの推定

$$\hat{a}_j^m = \hat{a}_j^{m-1} - \hat{a}_m^m \hat{a}_{m-j}^{m-1} \quad (j = 1, \ldots, m-1)$$

$$y_n = \sum_{i=1}^{m} a_j y_{n-j} + v_n$$

$$\hat{a}_m^m = (\hat{\sigma}_{m-1}^2)^{-1} \left\{ \hat{C}_m - \sum_{j=1}^{m-1} \hat{a}_j^{m-1} \hat{C}_{m-j} \right\}$$

⇒ Levinson's algorithm

$\hat{a}_m^m$をデータから直接推定する

$$y_n = \sum_{i=1}^{m-1} a_j^{m-1} y_{n+j} + w_n^{m-1}$$

$$\begin{aligned}
C_m - \sum_{j=1}^{m-1} a_j^{m-1} C_{m-j} &= E\left\{ \left(y_n - \sum_{i=1}^{m-1} a_j^{m-1} y_{n-j} \right) y_{n-m} \right\} \\
&= E\left(v_n^{m-1} y_{n-m} \right) \\
&= E\left(v_n^{m-1} w_{n-m}^{m-1} \right) \\
&\cong \frac{1}{N-m} \sum_{n=m+1}^{N} \left(v_n^{m-1} w_{n-m}^{m-1} \right)
\end{aligned}$$

（４）PARCOR法によるARモデルの推定

$$C_0 - \sum_{j=1}^{m-1} a_j^{m-1} C_j = E\left\{\left(y_{n-m} - \sum_{i=1}^{m-1} a_j^{m-1} y_{n-m+j} \right) y_{n-m}\right\}$$

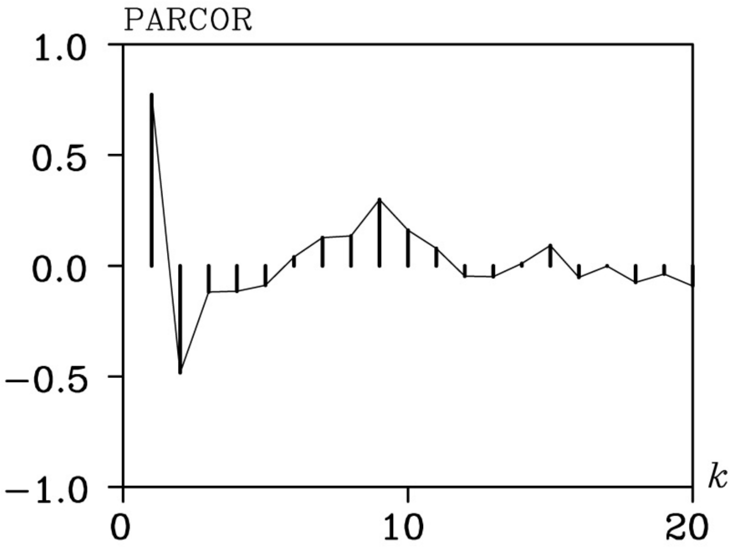
$$= E\left(w_{n-m}^{m-1} y_{n-m} \right)$$

$$= E\left(w_{n-m}^{m-1} \right)^2$$

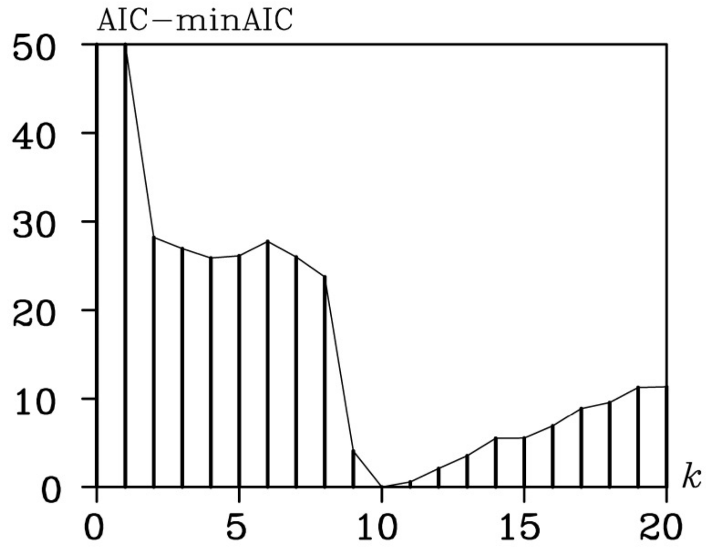
$$\cong \begin{cases} \dfrac{1}{N-m} \displaystyle\sum_{n=m+1}^{N} \left(w_{n-m}^{m-1} \right)^2 \\ \dfrac{1}{N-m} \left\{ \displaystyle\sum_{n=m+1}^{N} \left(w_{n-m}^{m-1} \right)^2 \sum_{n=m+1}^{N} \left(v_n^{m-1} \right)^2 \right\}^{\frac{1}{2}} \\ \dfrac{1}{2(N-m)} \left\{ \displaystyle\sum_{n=m+1}^{N} \left(w_{n-m}^{m-1} \right)^2 + \sum_{n=m+1}^{N} \left(v_n^{m-1} \right)^2 \right\} \end{cases}$$

$$a_m^m = \begin{cases} \displaystyle\sum_{n=m+1}^{N} v_n^{m-1} w_{n-m}^{m-1} \left\{ \sum_{n=m+1}^{N} \left(w_{n-m}^{m-1} \right)^2 \right\}^{-1} \\ \displaystyle\sum_{n=m+1}^{N} v_n^{m-1} w_{n-m}^{m-1} \left\{ \sum_{n=m+1}^{N} \left(w_{n-m}^{m-1} \right)^2 \sum_{n=m+1}^{N} \left(v_n^{m-1} \right)^2 \right\}^{-\frac{1}{2}} \\ \displaystyle\sum_{n=m+1}^{N} v_n^{m-1} w_{n-m}^{m-1} \left\{ \sum_{n=m+1}^{N} \left(w_{n-m}^{m-1} \right)^2 + \sum_{n=m+1}^{N} \left(v_n^{m-1} \right)^2 \right\}^{-1} \end{cases}$$

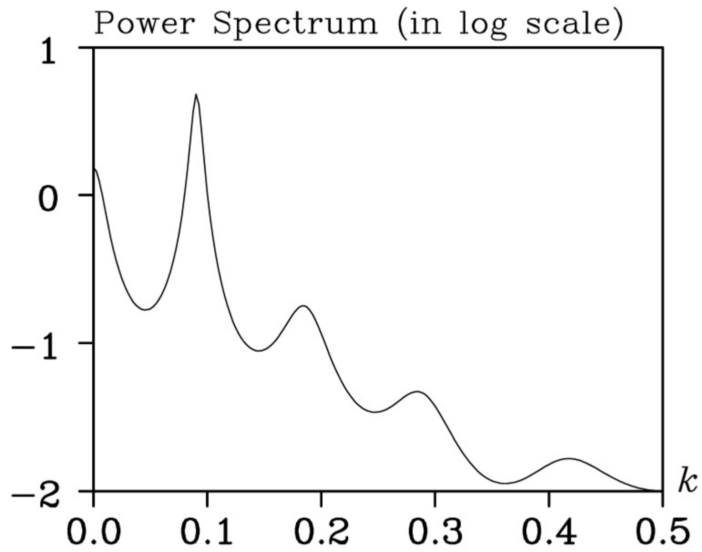
数値例　Candian Lynx data



(a)



(b)



(c)

```
# AR model fitting for Candian Lynx data
# method=1 (default) Yule-Walker method
# method=2 Householder least squares method
# method=3 Parcor method (Partial autoregrssion)
# method=4 PARCOR
# method=5 Burg's algorithm (MEM)

arfit(lynx,method=2)
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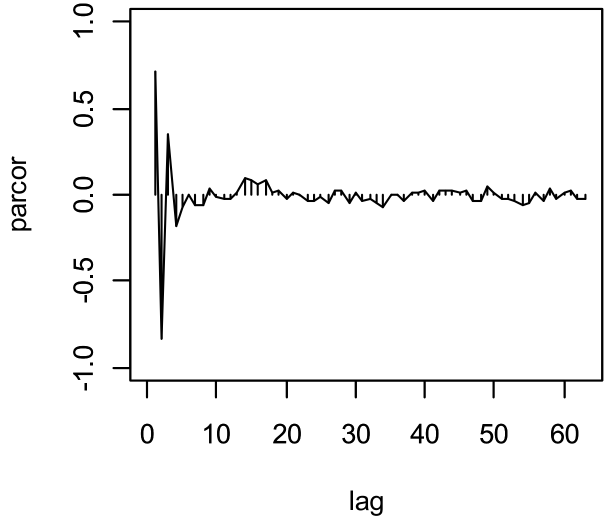
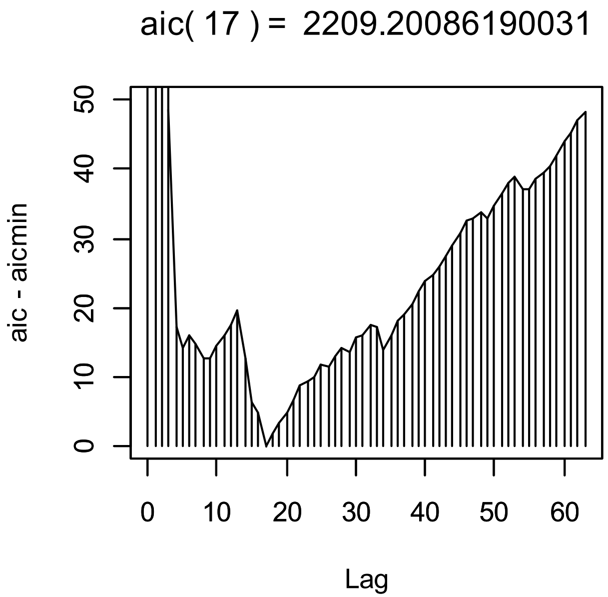
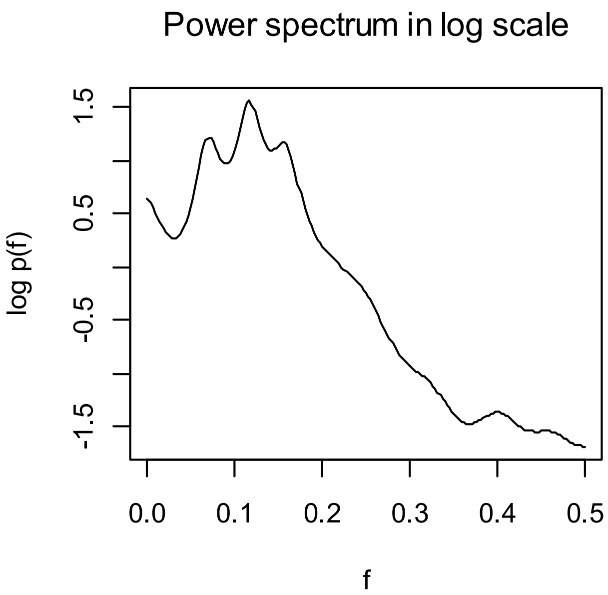
Hakusan (ship) data

arfit(hakusan1)

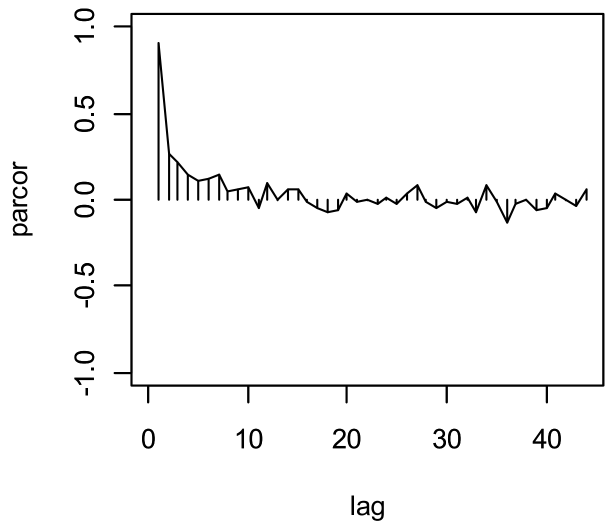


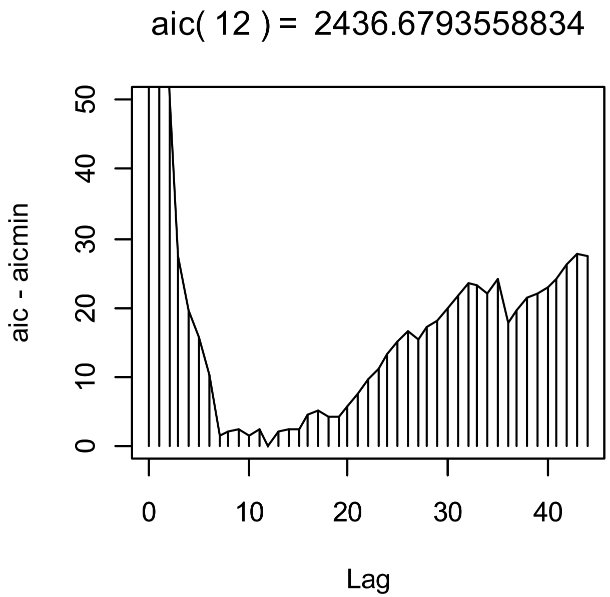


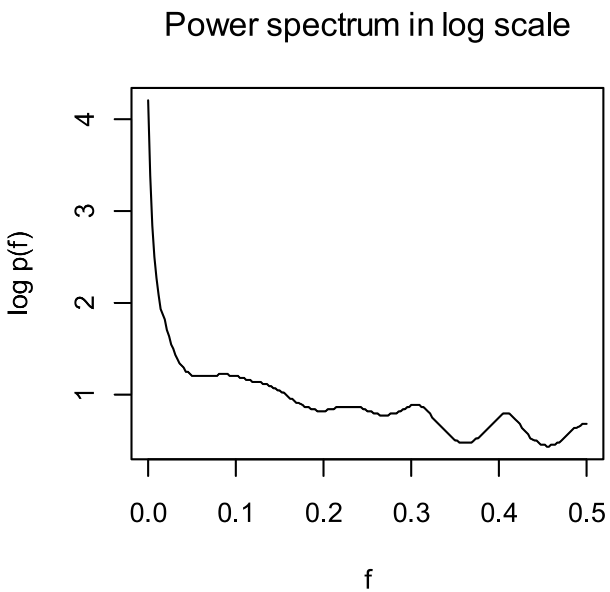


Maximum temperature data

arfit(maxtemp)

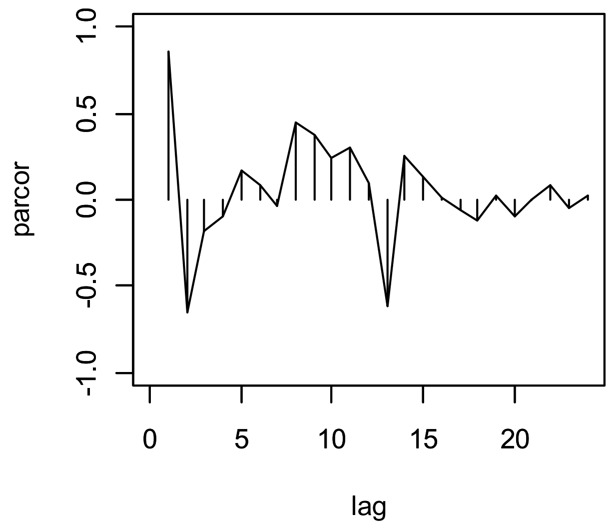


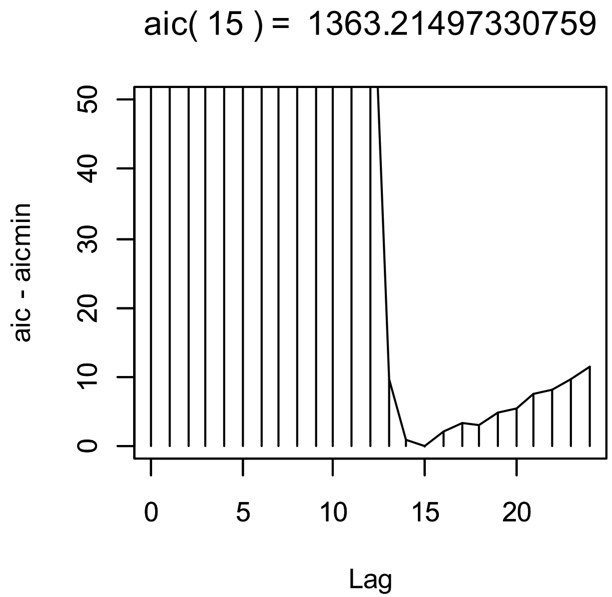


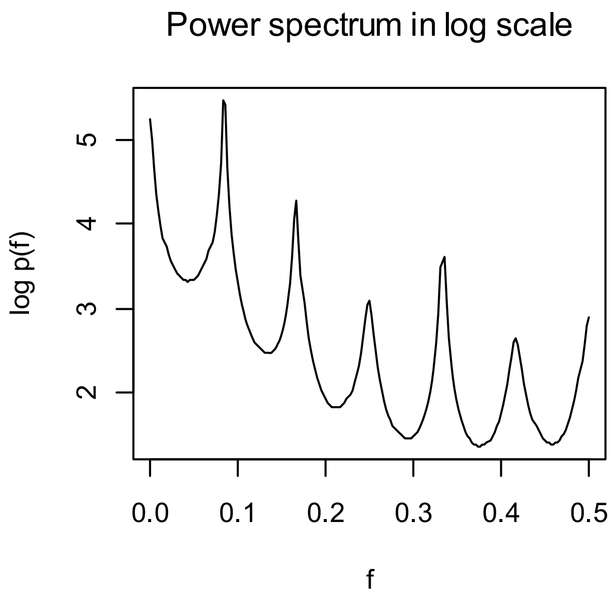


BLSFOOD data

arfit(blsfood)

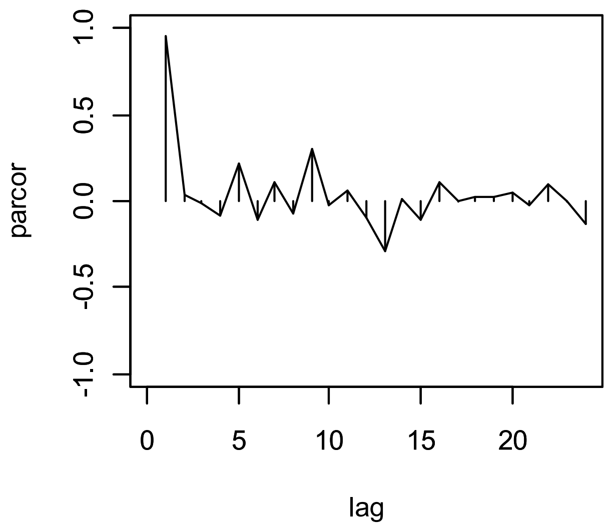


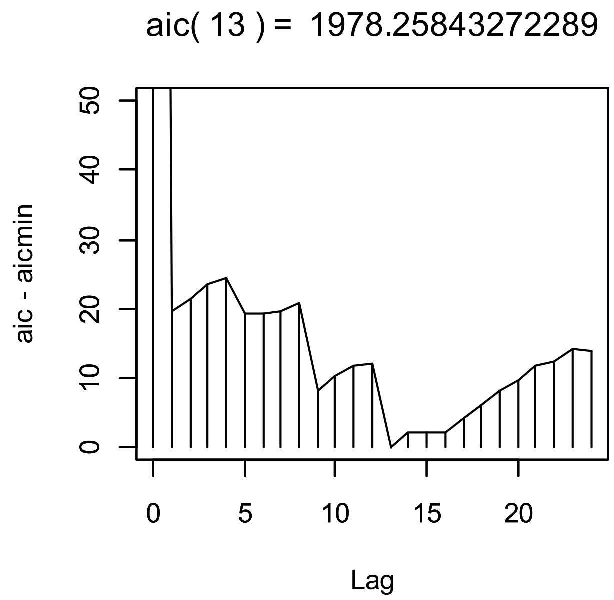


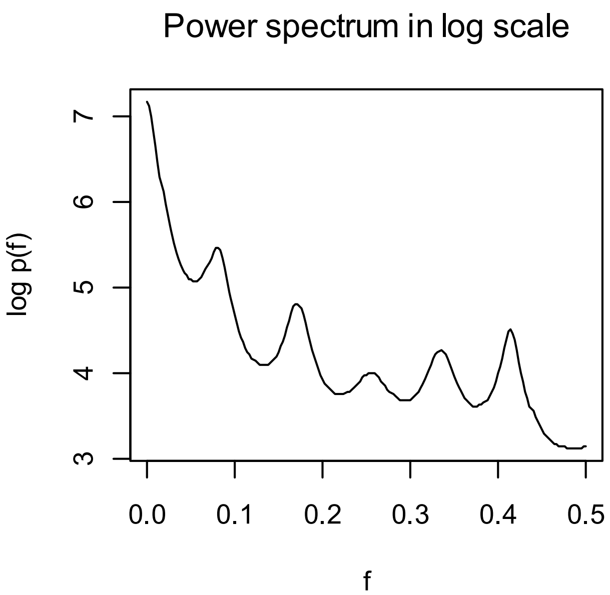


WHARD data

arfit(whard)

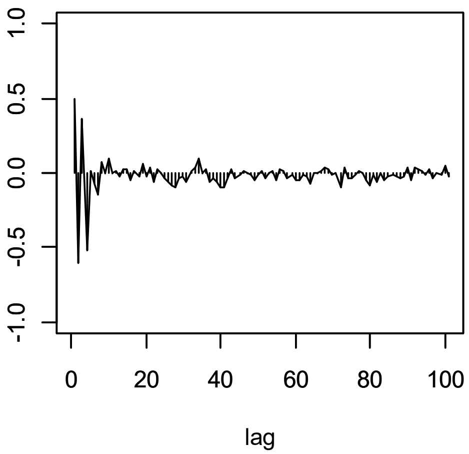


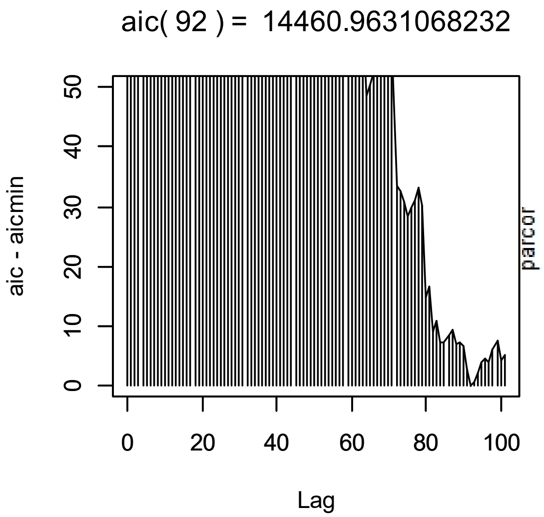


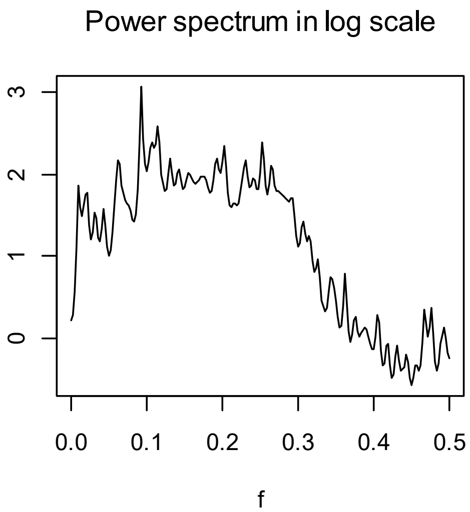


MYE1F earthquake data

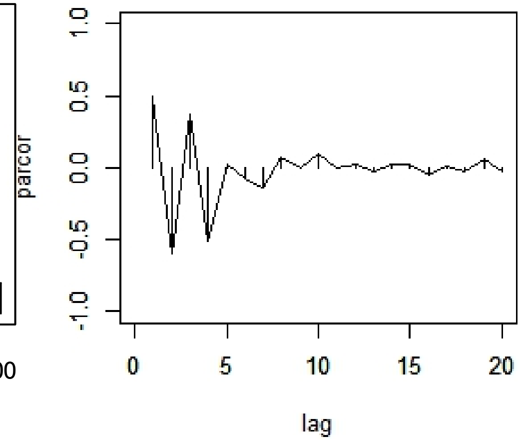
arfit(mye1f)

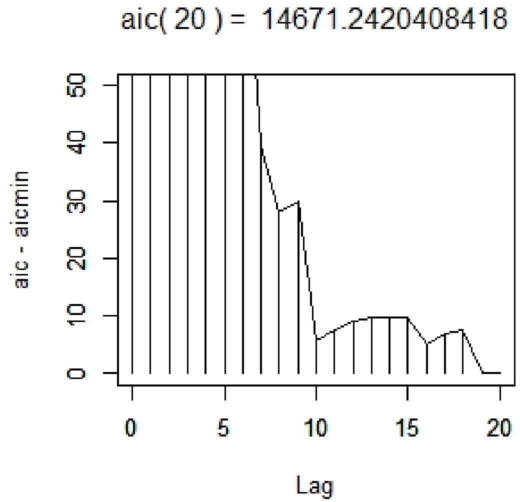


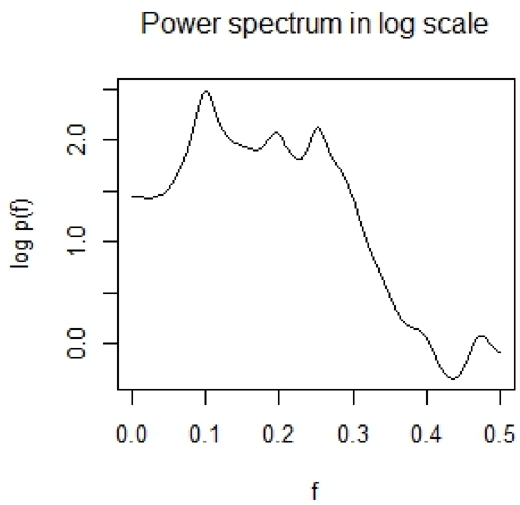




arfit(mye1f,lag=20)







多変量ARモデルの推定（Householder法）

$$S=\left[\begin{array}{ccc|c} s_{11} & \cdots & s_{1,km} & s_{1,km+1} \\ & \ddots & \vdots & \vdots \\ & & s_{km,km} & s_{km,km+1} \\ & & & s_{km+1,km+1} \end{array}\right] \quad km+1$$

$$\begin{array}{c} \overset{km+1}{\longleftrightarrow} \\ \overset{kj}{\longleftrightarrow} \end{array}$$

$$S=\begin{bmatrix} s_{11} & \cdots & s_{1,kj} & \cdots & s_{1,km} & s_{1,km+1} \\ & \ddots & \vdots & & \vdots & \vdots \\ & & s_{kj,kj} & \cdots & s_{kj,km} & s_{kj,km+1} \\ & & & \ddots & \vdots & \vdots \\ & & & & s_{km,km} & s_{km,km+1} \\ & & & & & s_{km+1,km+1} \end{bmatrix}$$

$$y_n(1)=\sum_{i=1}^{j} b_i(1,1)y_{n-i}(1)+\cdots+\sum_{i=1}^{j} b_i(1,k)y_{n-i}(k)+w_n$$

$$\hat{\sigma}_j^2(1)=\frac{1}{N-m}\sum_{i=kj+1}^{km+1} s_{i,km+1}^2$$

$$\mathrm{AIC}_j(1)=(N-m)\log(2\pi\hat{\sigma}_j^2(1)+1)+2(kj+1)$$

$$\begin{bmatrix} s_{11} & \cdots & s_{1,kj} \\ & \ddots & \vdots \\ & & s_{kj,kj} \end{bmatrix}\begin{bmatrix} c_1 \\ \vdots \\ c_{kj} \end{bmatrix}=\begin{bmatrix} s_{1,km+1} \\ \vdots \\ s_{kj,km+1} \end{bmatrix}$$

$$y_n(2) = \sum_{i=1}^{j} b_i(2,1) y_{n-i}(1) + \cdots + \sum_{i=1}^{j} b_i(2,k) y_{n-i}(k) + w_n$$

$$S = \begin{bmatrix} s_{11} & \cdots & s_{1,km} & s_{1,km+1} & s_{1,km+2} & \cdots & s_{1,km+k} \\ s_{21} & \cdots & s_{2,km} & & s_{2,km+2} & \cdots & s_{2,km+k} \\ & \ddots & \vdots & & \vdots & & \vdots \\ & & s_{km+1,km} & & s_{km+1,km+2} & \cdots & s_{km+1,km+k} \\ & & & & s_{km+2,km+2} & \cdots & s_{km+1,km+k} \\ & & & & & \ddots & \vdots \\ & & & & & & s_{km+k,km+k} \end{bmatrix}$$

$$\hat{\sigma}_j^2(2) = \frac{1}{N-m} \sum_{i=kj+2}^{km+2} s_{i,km+2}^2$$

$$\mathrm{AIC}_j(2) = (N-m)\log(2\pi\hat{\sigma}_j^2(2)+1) + 2(kj+2)$$

$$\begin{bmatrix} s_{11} & \cdots & s_{1,kj} & s_{1,km+1} \\ s_{21} & \cdots & s_{2,kj} & \\ & \ddots & \vdots & \\ & & s_{kj+1,kj} & \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{kj+1} \end{bmatrix} = \begin{bmatrix} s_{1,km+2} \\ s_{2,km+2} \\ \vdots \\ s_{kj+1,km+2} \end{bmatrix}$$