時系列解析（５）

－ARMAモデルによる時系列の解析－

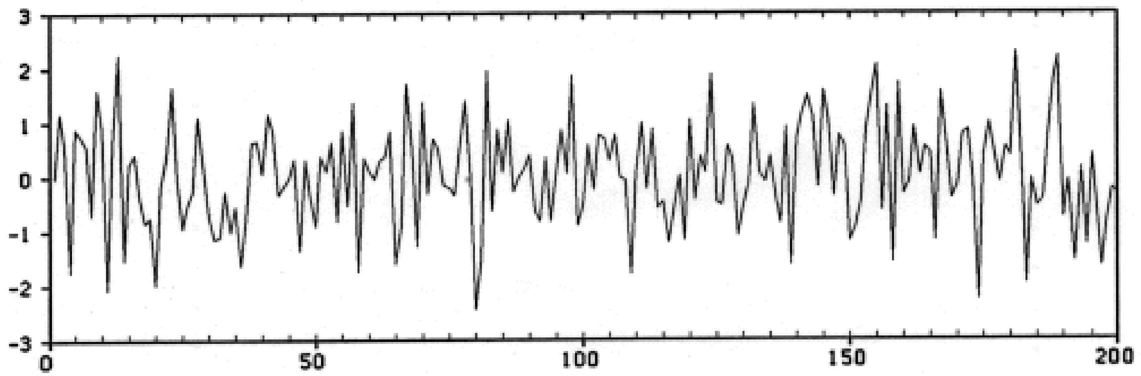
配布用

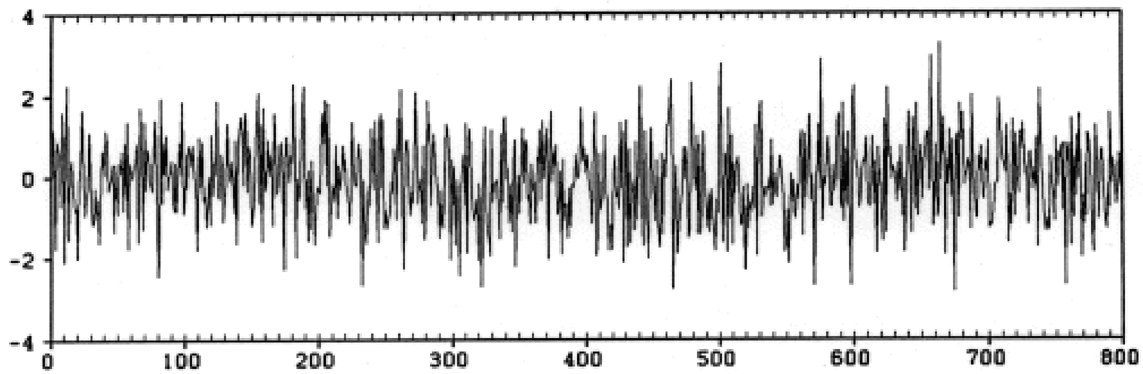
東京大学 数理・情報教育研究センター

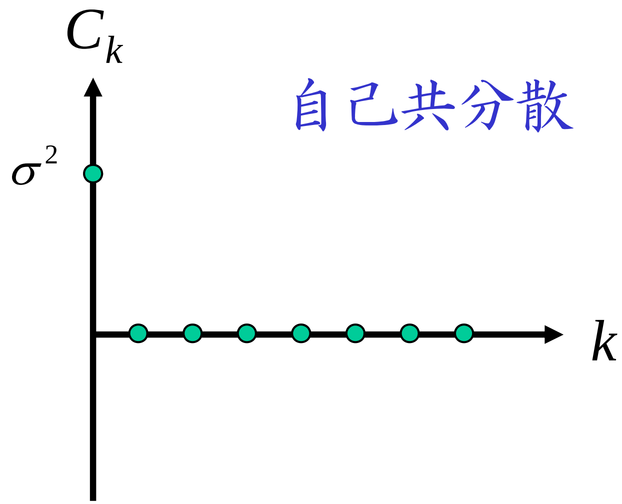
北川 源四郎

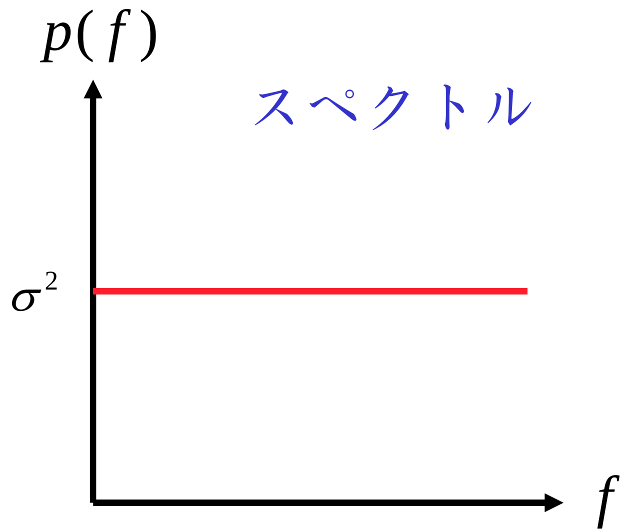
白色雑音

白色雑音の例







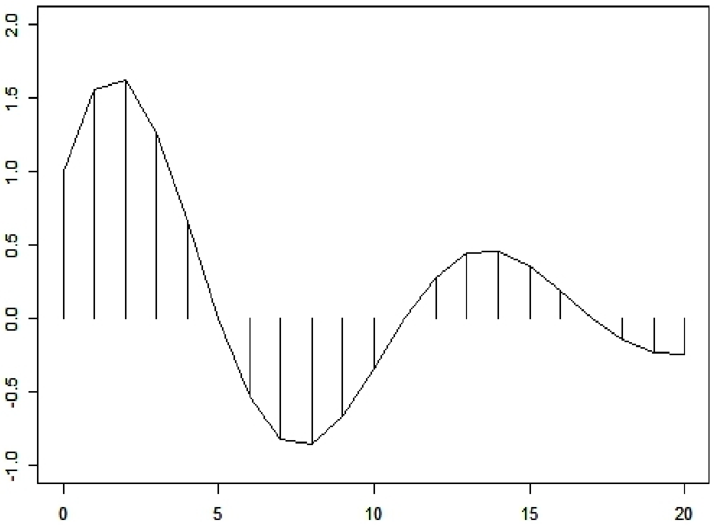


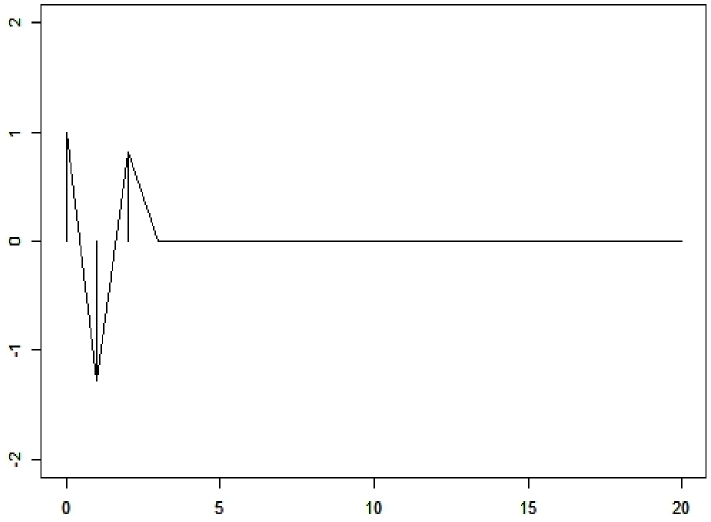
インパルス応答関数

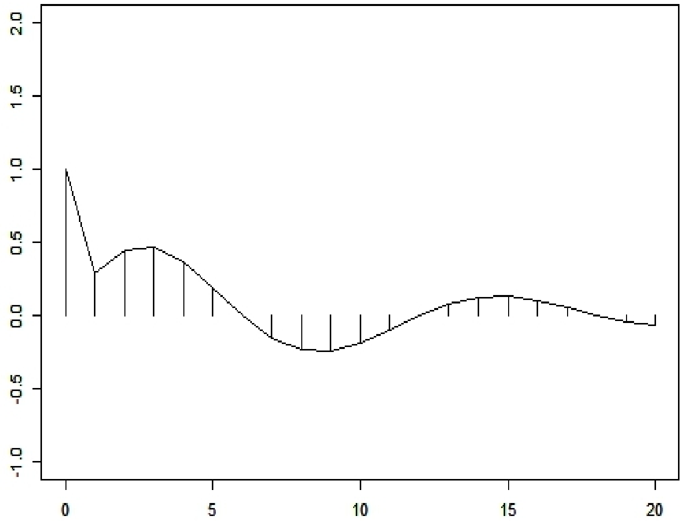
(1) $$y_n = 0.9\sqrt{3}y_{n-1} - 0.81y_{n-2} + v_n$$

(2) $$y_n = v_n - 0.9\sqrt{2}v_{n-1} + 0.81v_{n-2}$$

(3) $$y_n = 0.9\sqrt{3}y_{n-1} - 0.81y_{n-2} + v_n - 0.9\sqrt{2}v_{n-1} + 0.81v_{n-2}$$





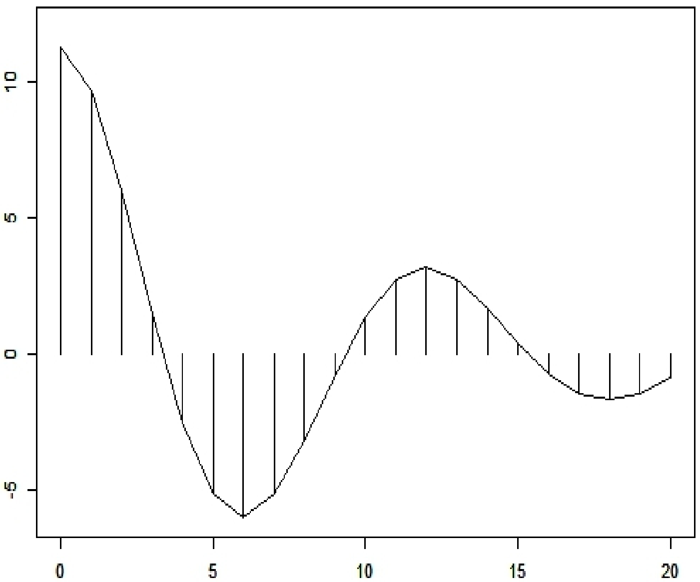


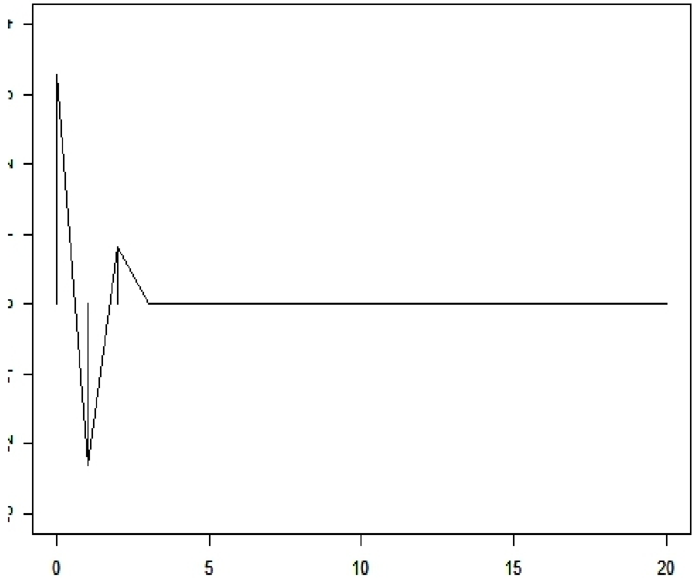
自己共分散関数

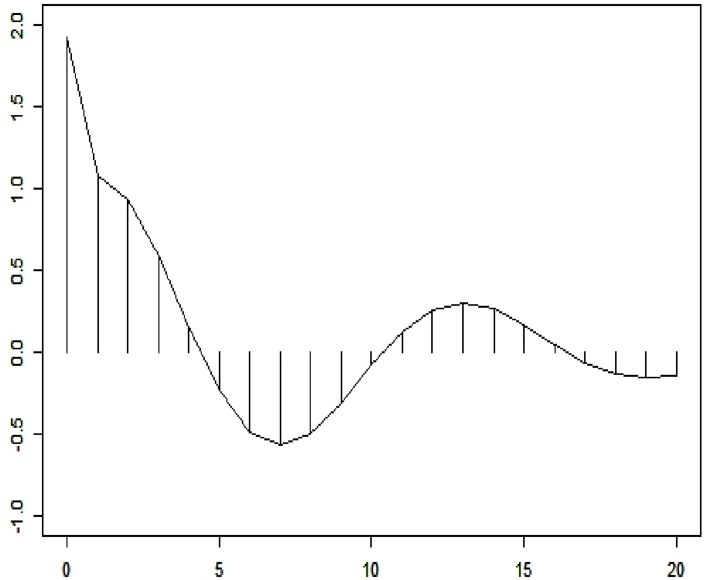
(1) $$y_n = 0.9\sqrt{3}y_{n-1} - 0.81y_{n-2} + v_n$$

(2) $$y_n = v_n - 0.9\sqrt{2}v_{n-1} + 0.81v_{n-2}$$

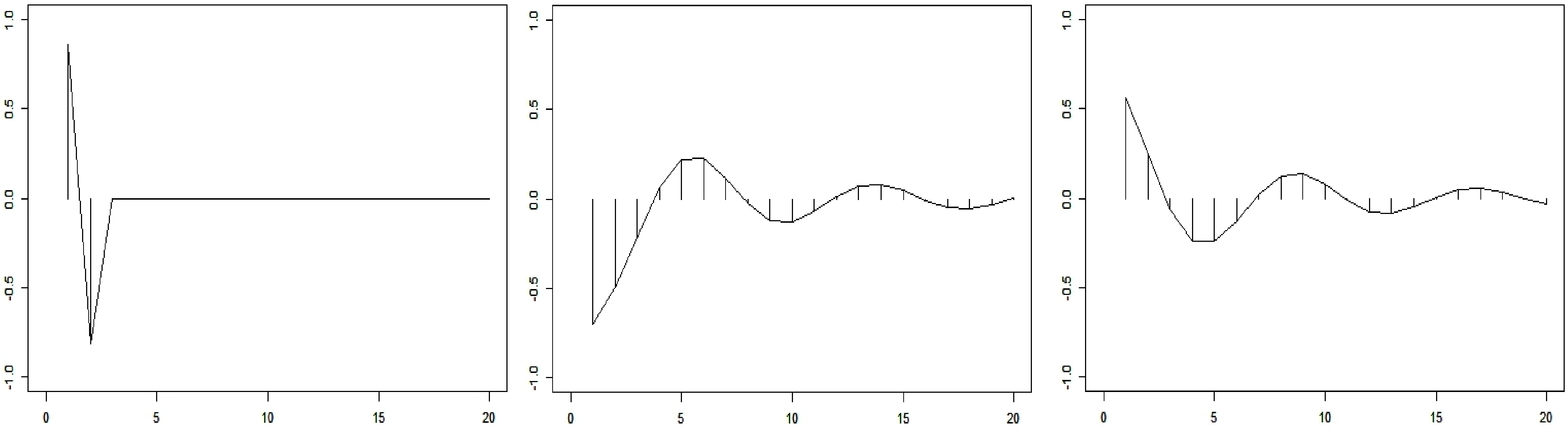
(3) $$y_n = 0.9\sqrt{3}y_{n-1} - 0.81y_{n-2} + v_n - 0.9\sqrt{2}v_{n-1} + 0.81v_{n-2}$$







偏自己相関係数 (PARCOR)



AR, MA, ARMAモデルの特徴

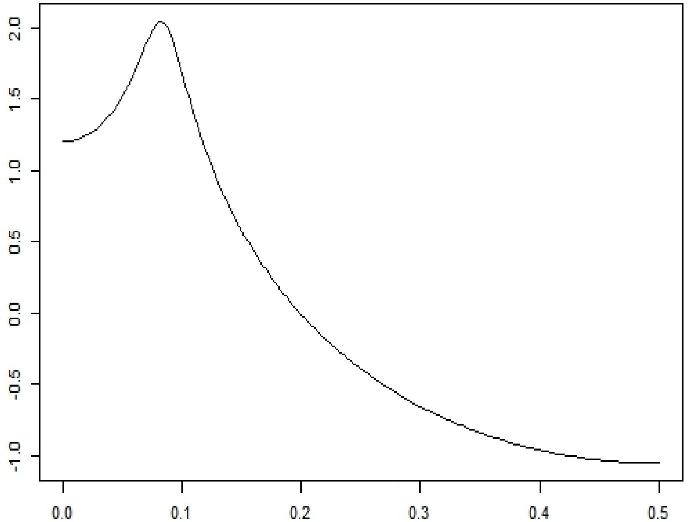
	C_k	b_k
AR(m)	∞	m
MA(ℓ)	ℓ	∞
ARMA(m,ℓ)	∞	∞

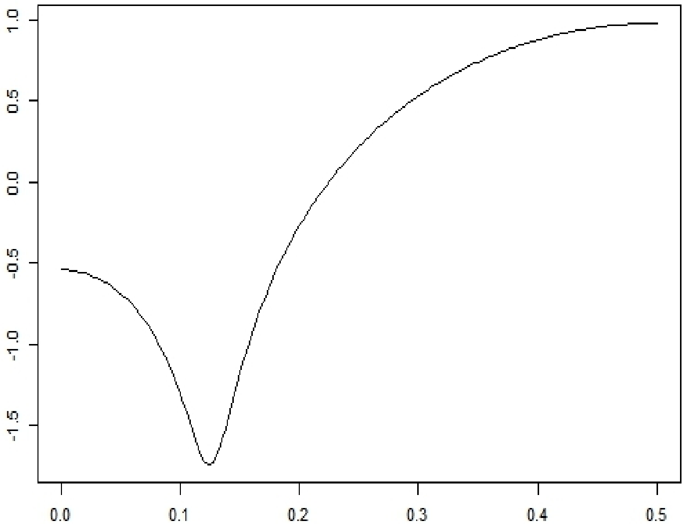
パワースペクトル

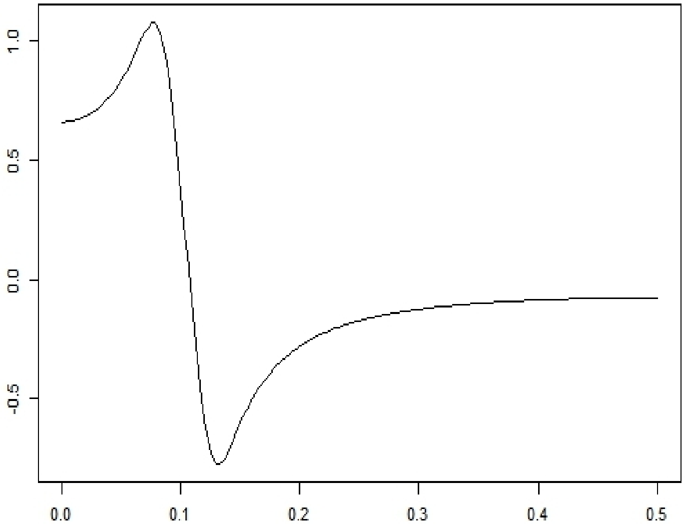
(1) $$y_n = 0.9\sqrt{3}y_{n-1} - 0.81y_{n-2} + v_n$$

(2) $$y_n = v_n - 0.9\sqrt{2}v_{n-1} + 0.81v_{n-2}$$

(3) $$y_n = 0.9\sqrt{3}y_{n-1} - 0.81y_{n-2} + v_n - 0.9\sqrt{2}v_{n-1} + 0.81v_{n-2}$$







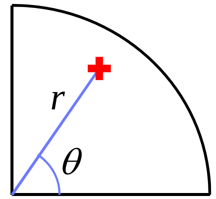
特性方程式

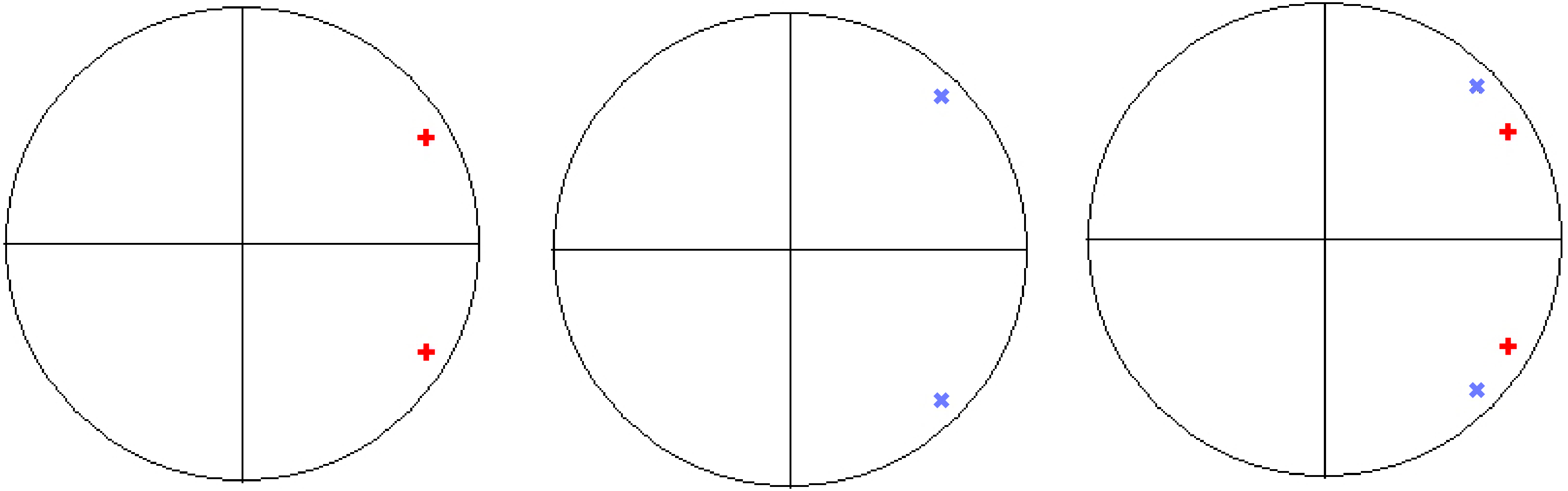
(1) $$y_n = 0.9\sqrt{3}y_{n-1} - 0.81y_{n-2} + v_n$$

(2) $$y_n = v_n - 0.9\sqrt{2}v_{n-1} + 0.81v_{n-2}$$

(3) $$y_n = 0.9\sqrt{3}y_{n-1} - 0.81y_{n-2} + v_n - 0.9\sqrt{2}v_{n-1} + 0.81v_{n-2}$$

$$z = \alpha + i\beta = re^{i\theta} \Rightarrow f = \frac{\theta}{2\pi}$$



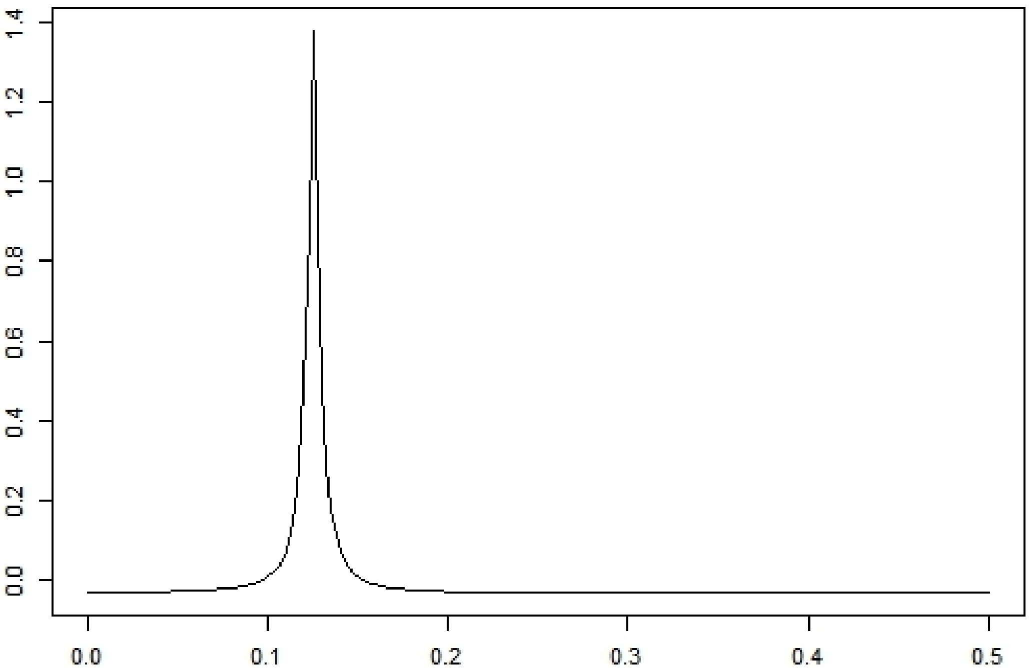


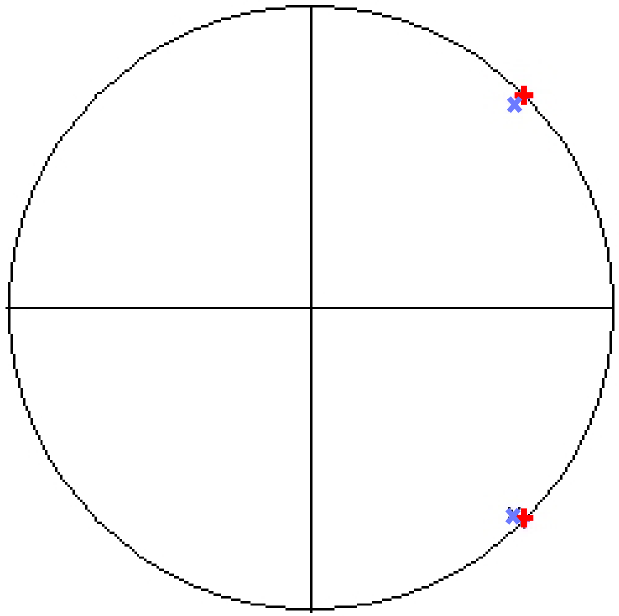
ラインスペクトル

$$y_n = a_1 y_{n-1} + a_2 y_{n-2} + v_n - b_1 v_{n-1} - b_2 v_{n-2}$$

$$a_1 = 0.99\sqrt{2} \quad a_2 = -0.99^2$$

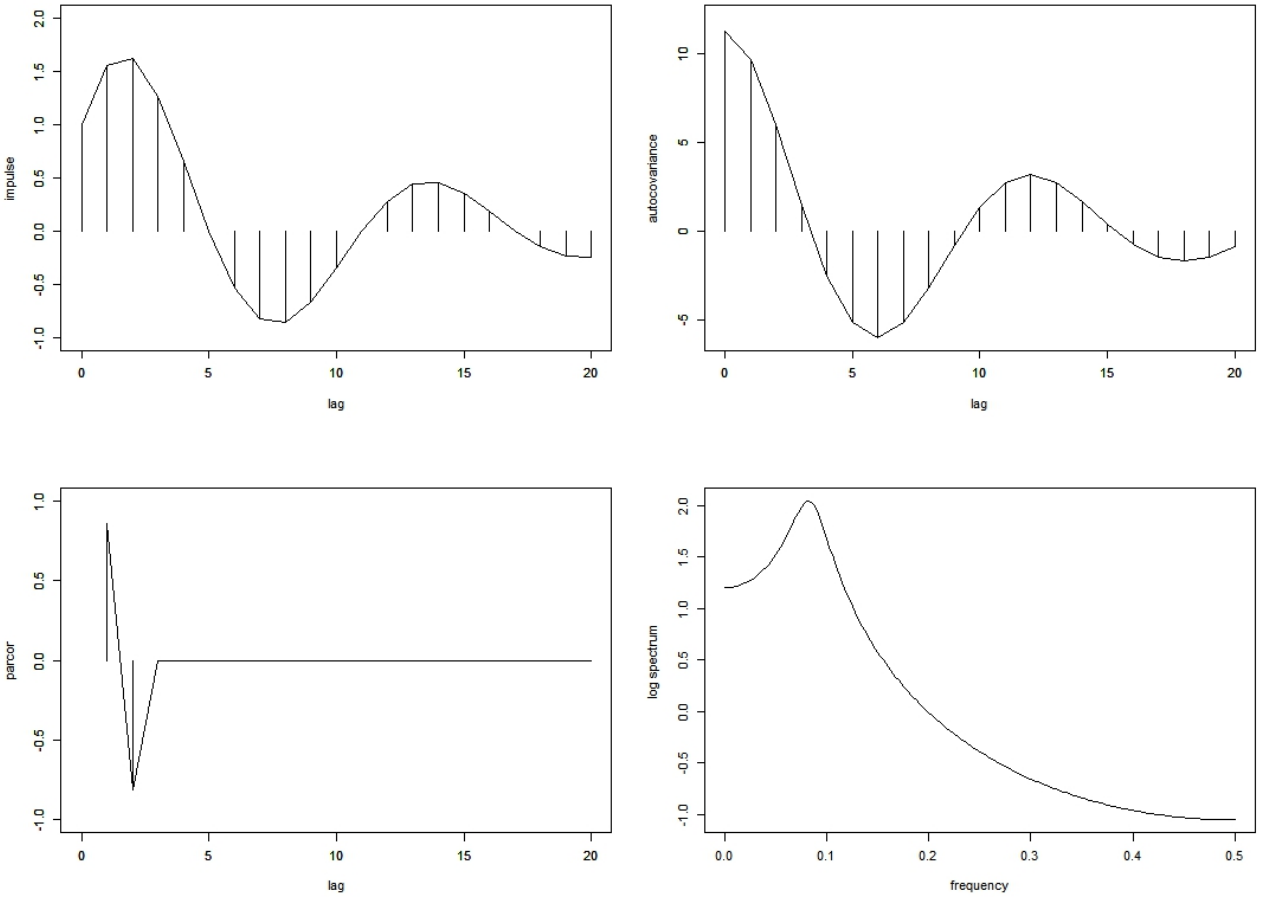
$$b_1 = 0.95\sqrt{2} \quad b_2 = -0.95^2$$

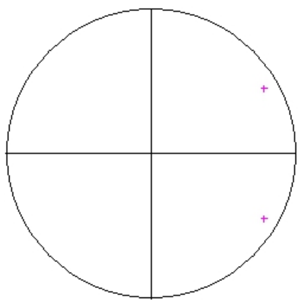




$$r = 0.99, \quad \theta = 45°$$
$$r = 0.95, \quad \theta = 45°$$

AR(2)モデル

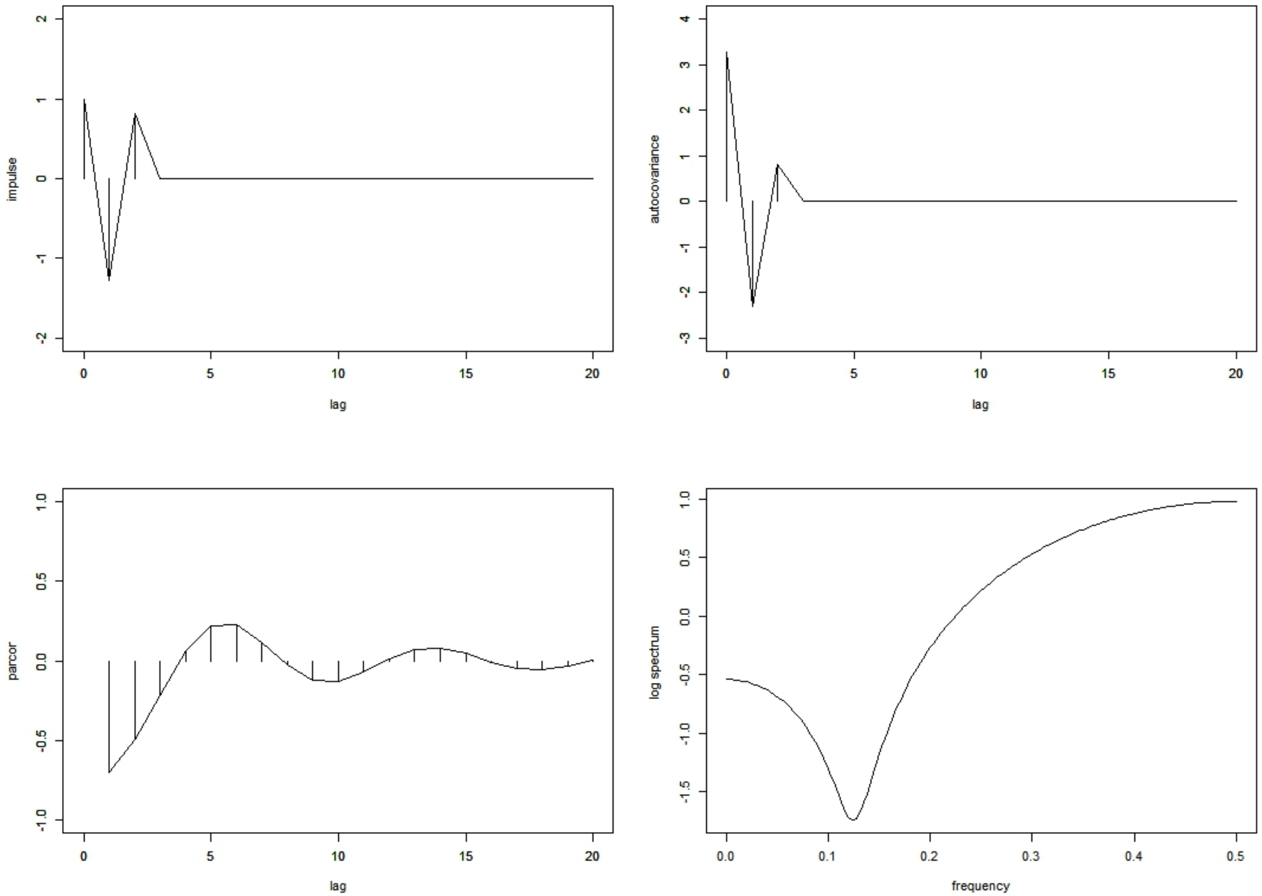


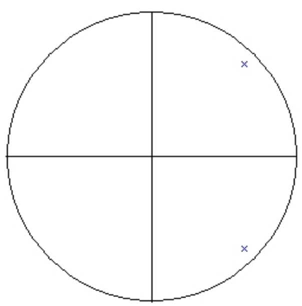


ARMA characteristic roots
(AR:pink, MA:blue)

```
# AR model : y(n) = 0.9sqrt(3)y(n-1) - 0.81y(n-2) + v(n)
z <- armaimp(arcoef=a, v=1.0, n=1000, lag=20)
z$croot.ar
```

MA(2)モデル

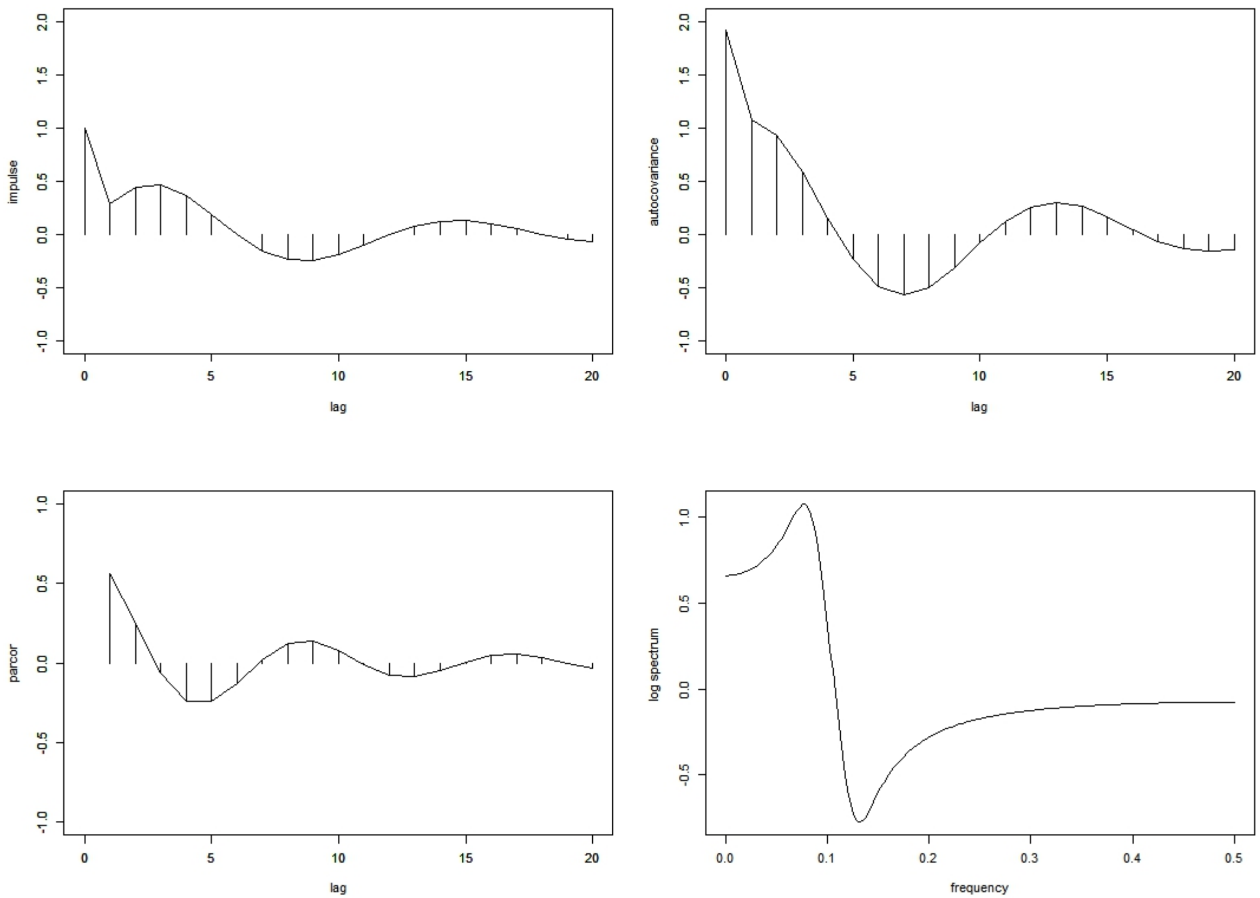


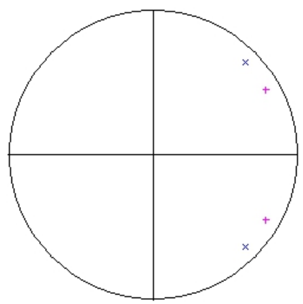


ARMA characteristic roots
(AR:pink, MA:blue)

```
# MA model : y(n) = v(n) -0.9sqrt(2)v(n-1) + 0.81v(n-2)
 z <- armaimp(macoef=b, v=1.0, n=1000, lag=20)
 z$croot.ma
```

ARMA(2,2)モデル

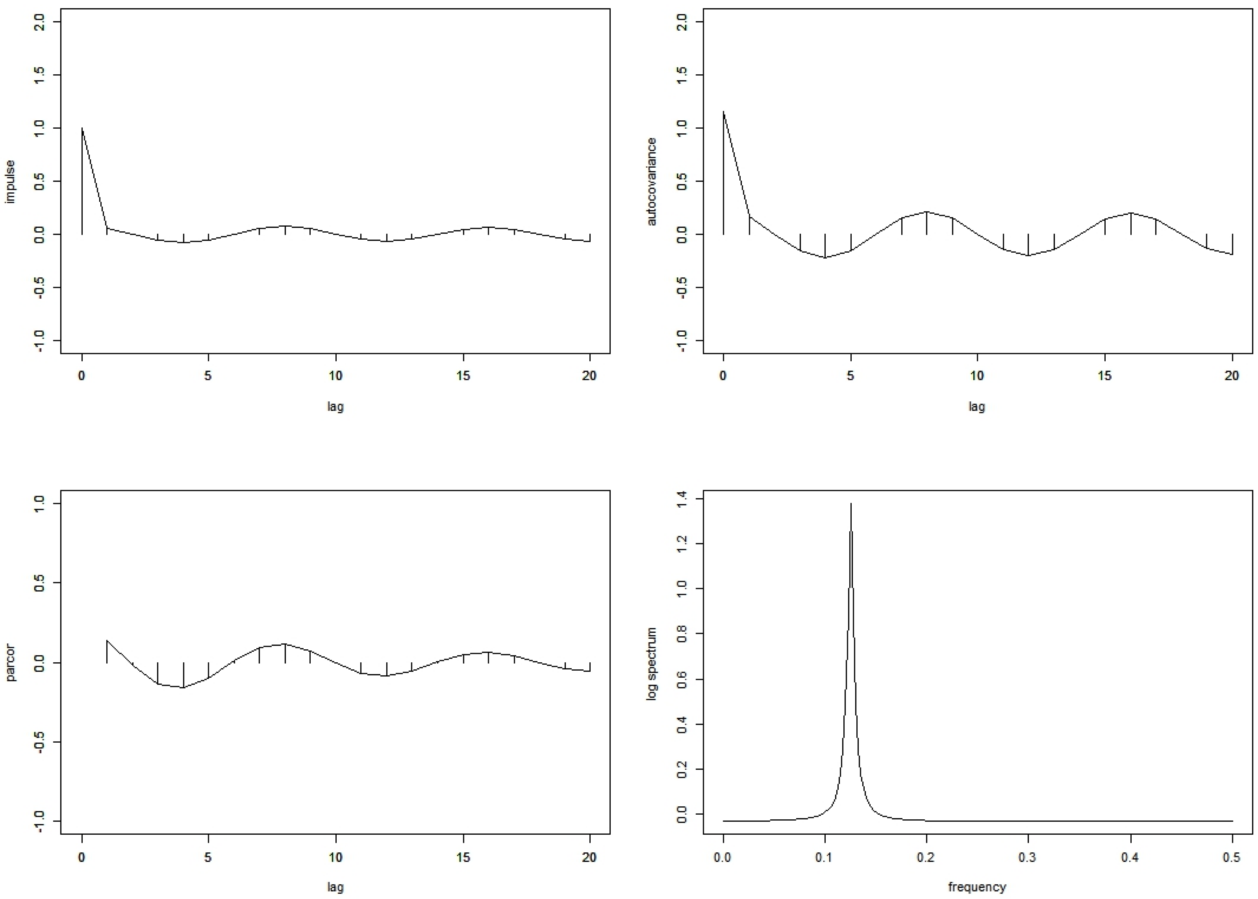


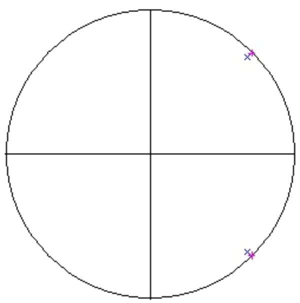


ARMA characteristic roots
(AR:pink, MA:blue)

```
# ARMA model :  y(n) = 0.9sqrt(3)y(n-1) - 0.81y(n-2)
 #                    + v(n) -0.9sqrt(2)v(n-1) + 0.81v(n-2)
 a <- c(0.9*sqrt(3), -0.81)
 b <- c(0.9*sqrt(2), -0.81)
 z <- armaimp(arcoef=a, macoef=b, v=1.0, n=1000, lag=20)
 z$croot.ar
 z$croot.ma
```

ARMA(2,2)モデル



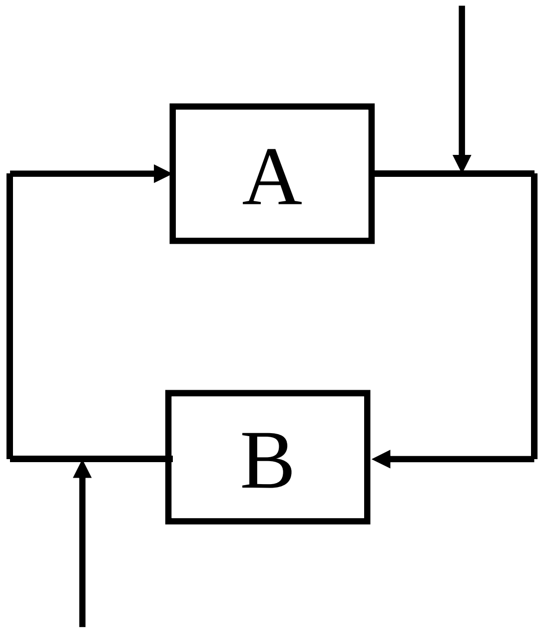


ARMA characteristic roots
(AR:pink, MA:blue)

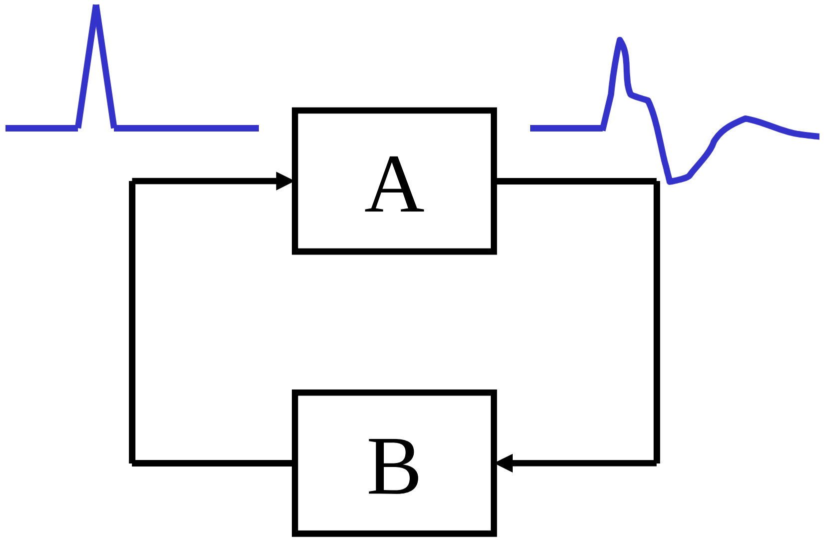
```
# ARMA model :  y(n) = 0.99sqrt(2)y(n-1) - 0.99^2y(n-2)
#                         + v(n) -0.95sqrt(2)v(n-1) + 0.95^2v(n-2)
a <- c(0.99*sqrt(2), -0.9801)
b <- c(0.95*sqrt(2), -0.9025)
z <- armaimp(arcoef=a, macoef=b, v=1.0, n=1000, lag=20)
z$croot.ar
z$croot.ma
```

インパルス応答関数

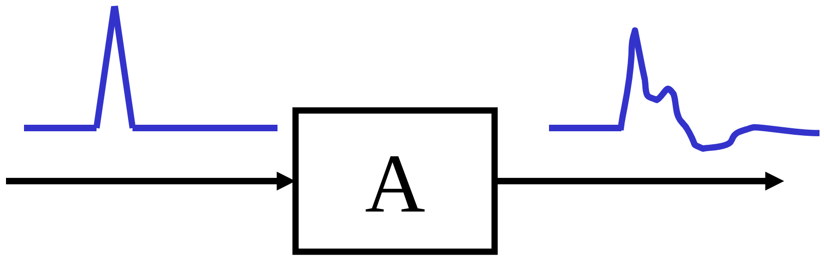
フィードバックシステム



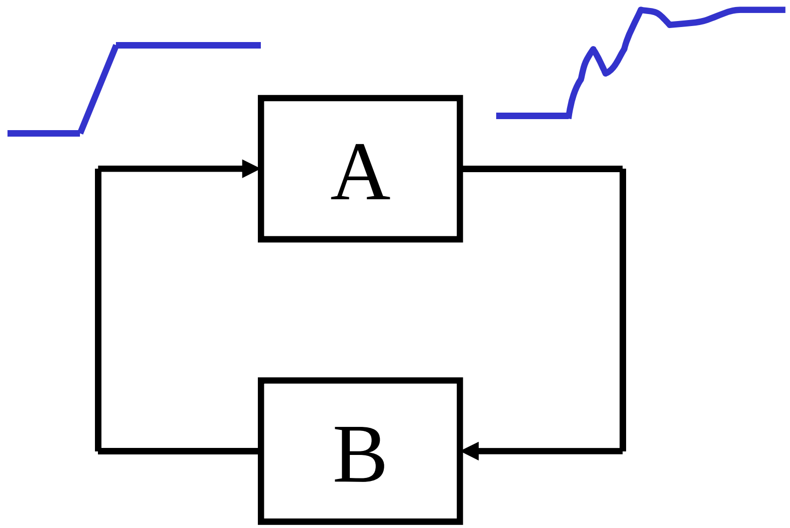
インパルス応答関数（開ループ）



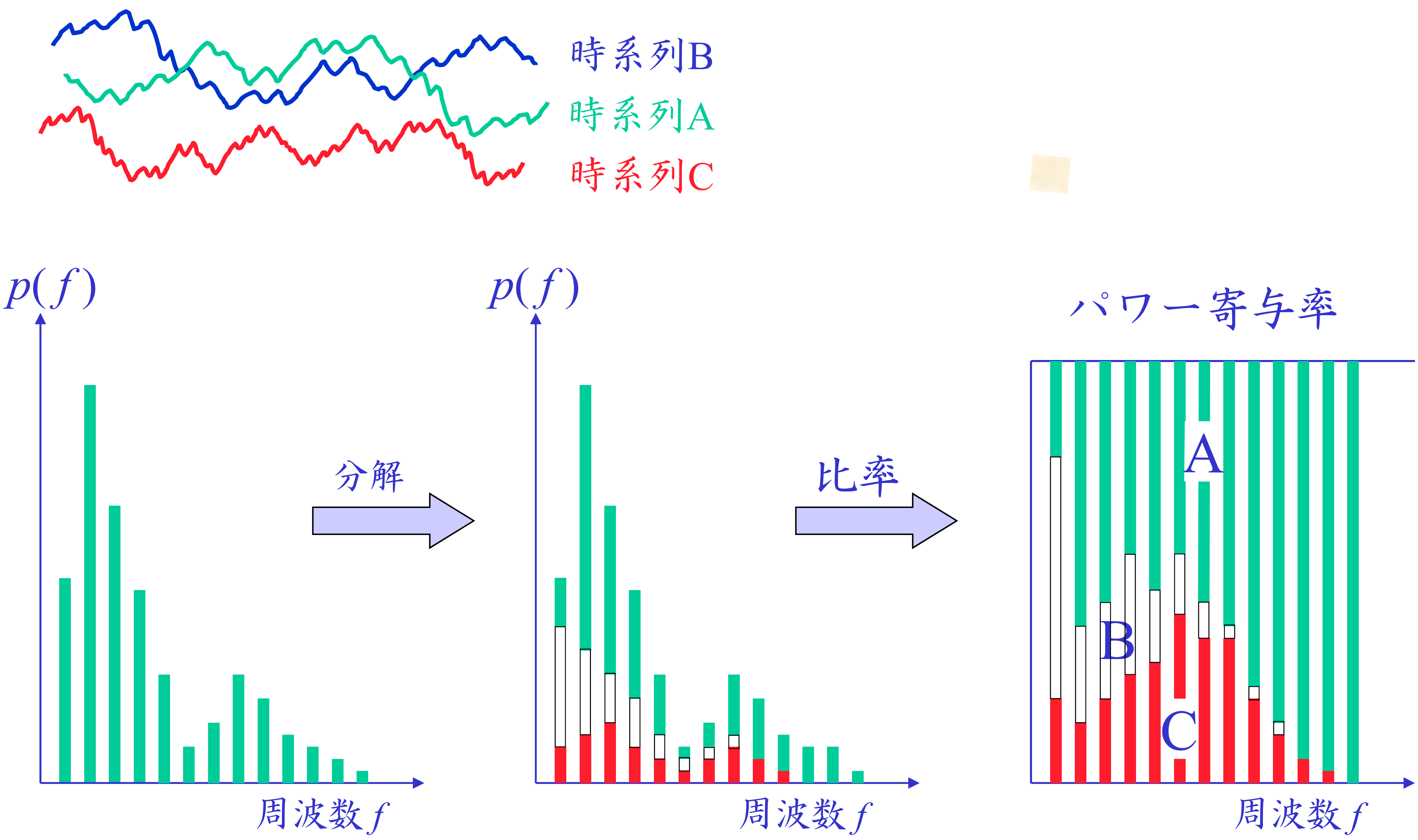
インパルス応答関数（開ループ）



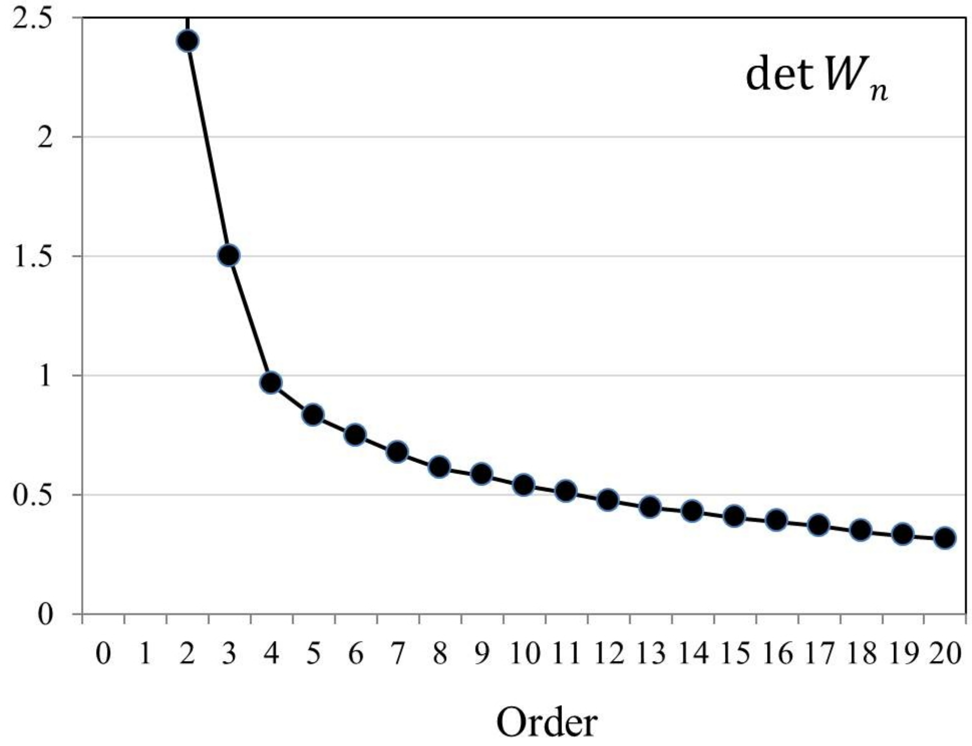
ステップ応答関数

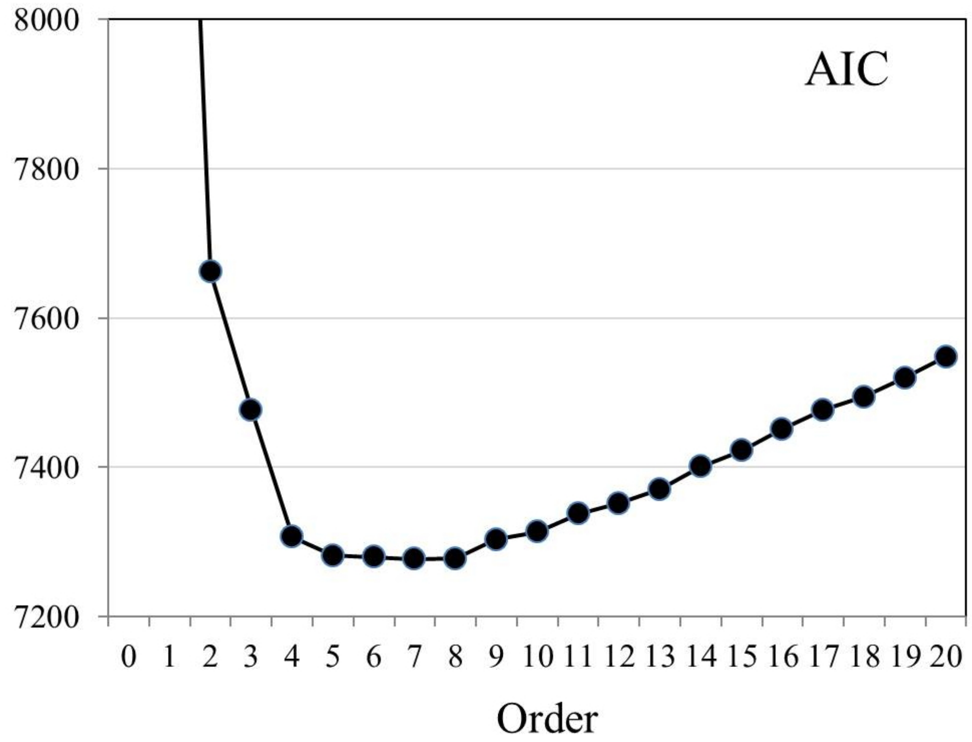


パワー寄与率

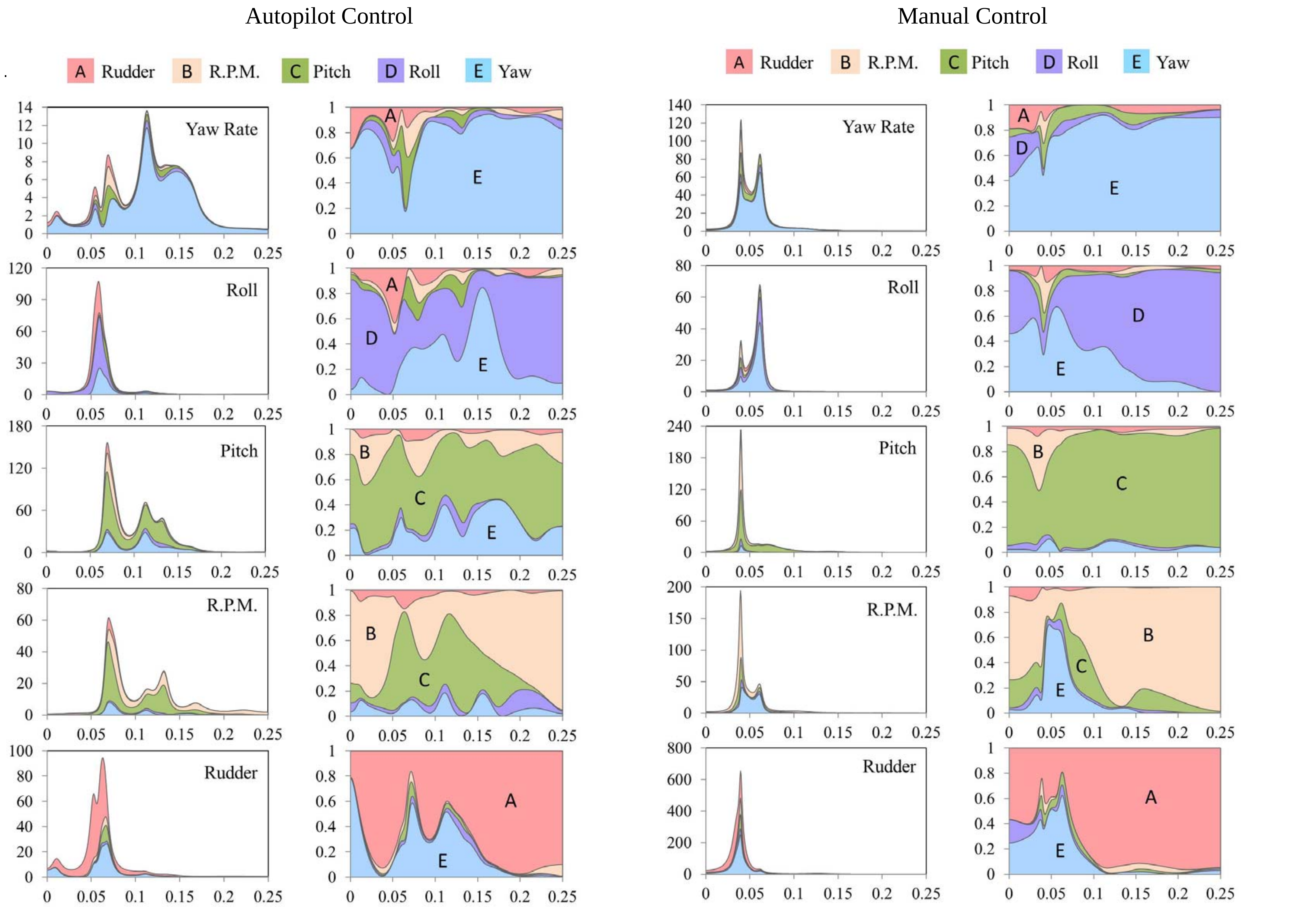


船体運動データ

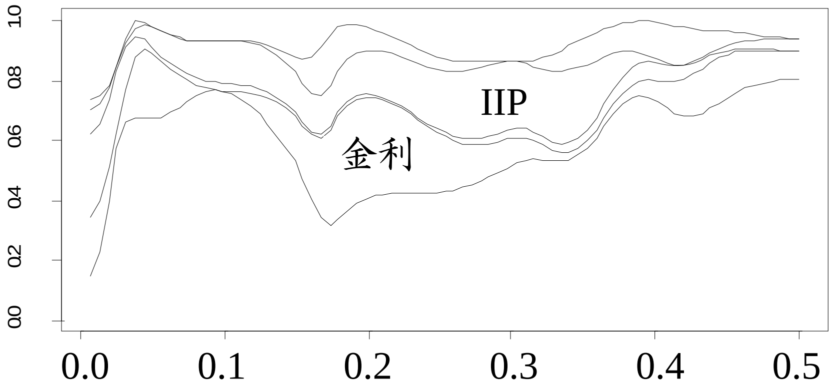




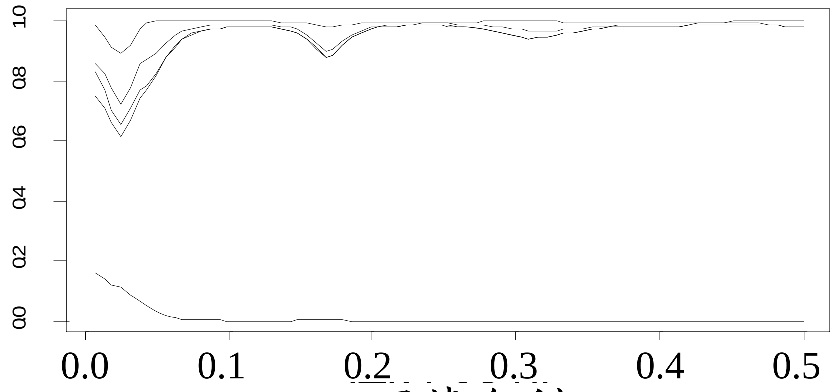
5変量（yaw rate, roll, pitch,rpm,rudder)


Autopilot Control
Manual Control
A Rudder
B R.P.M.
C Pitch
D Roll
E Yaw
Yaw Rate
Roll
Pitch
R.P.M.
Rudder
A Rudder
B R.P.M.
C Pitch
D Roll
E Yaw
Yaw Rate
Roll
Pitch
R.P.M.
Rudder

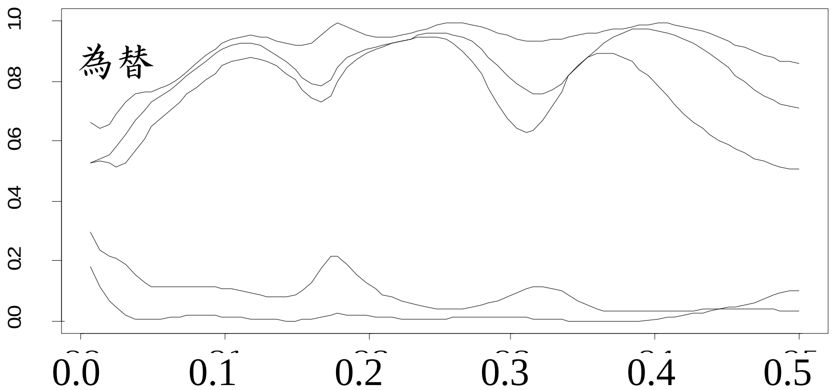
パワー寄与率



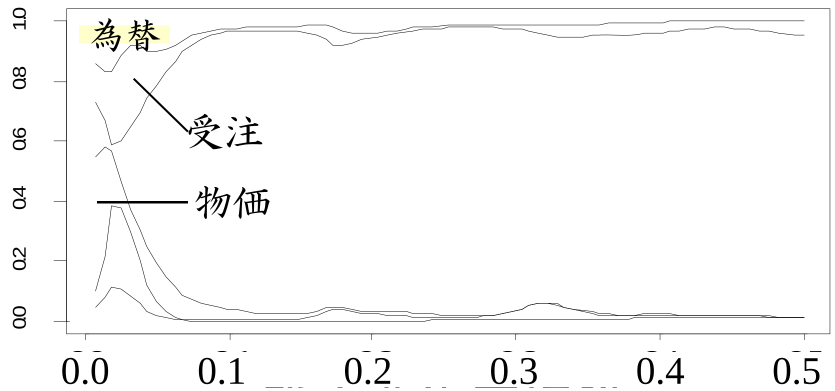
マネタリーベース



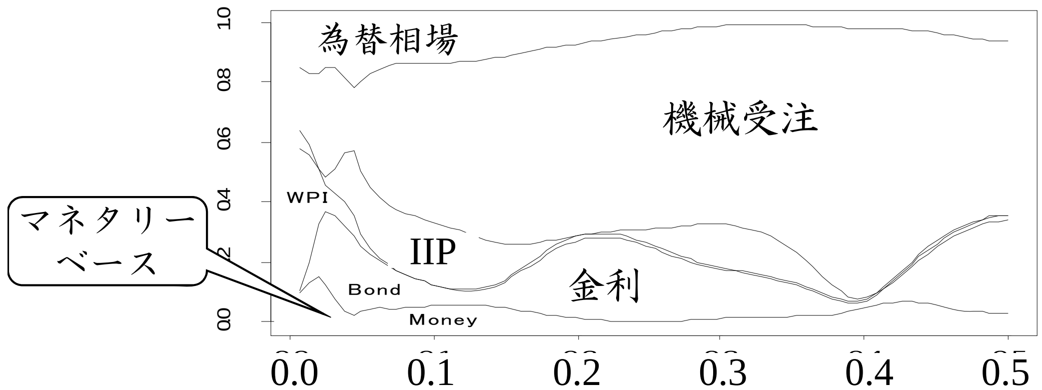
国債金利



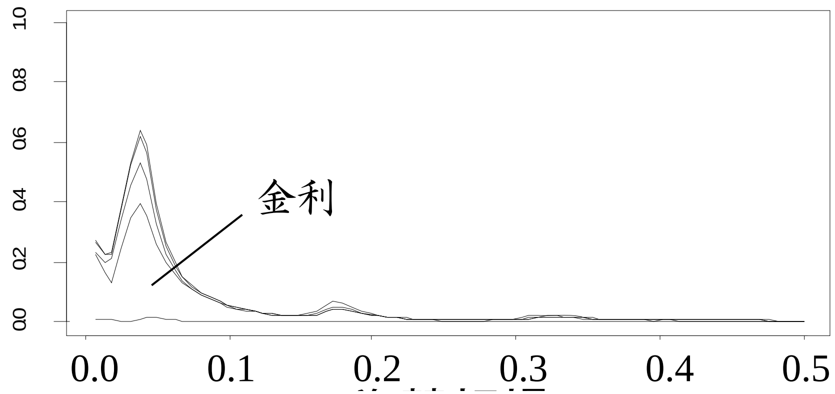
卸売物価



鉱工業生産指数

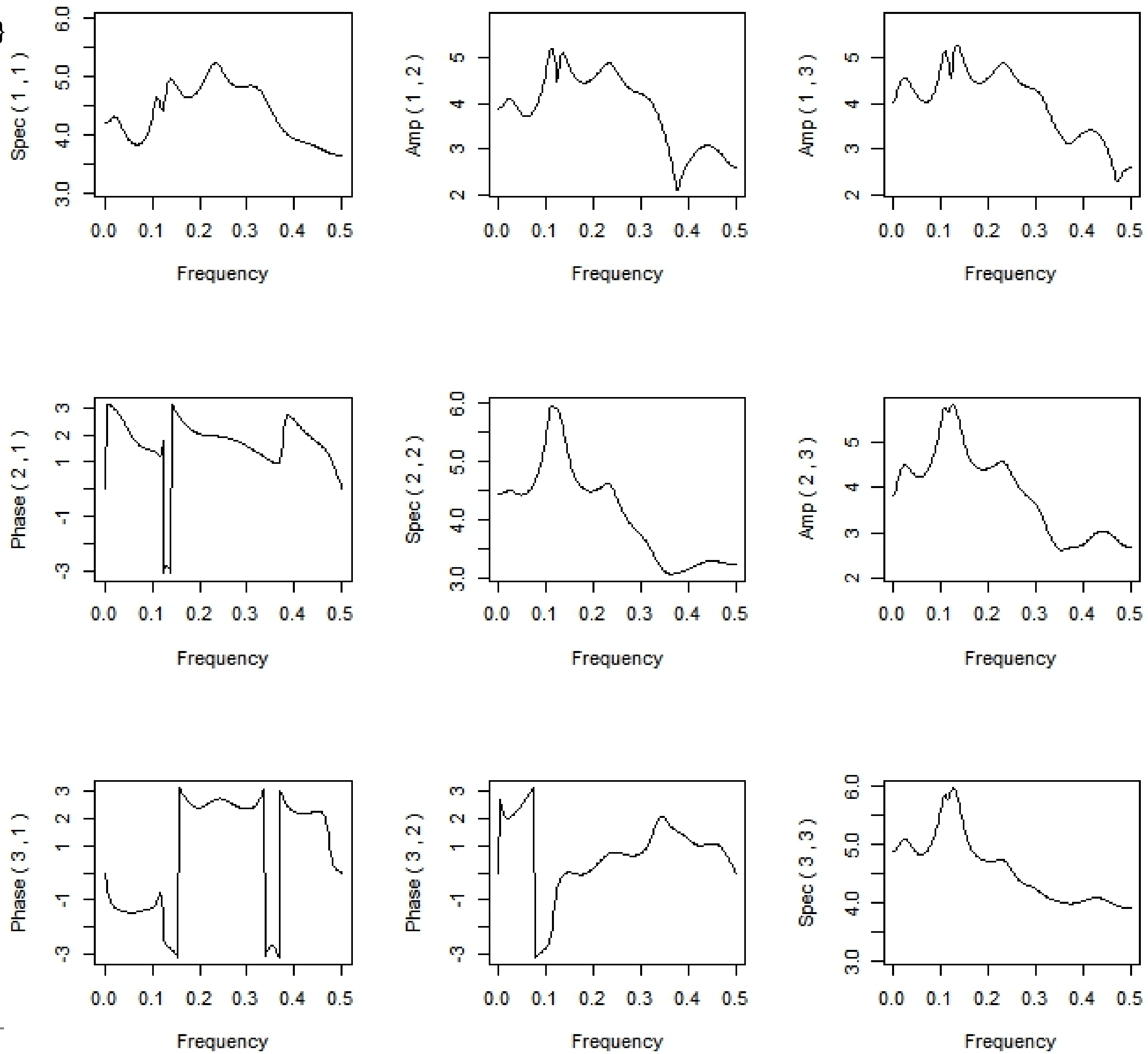


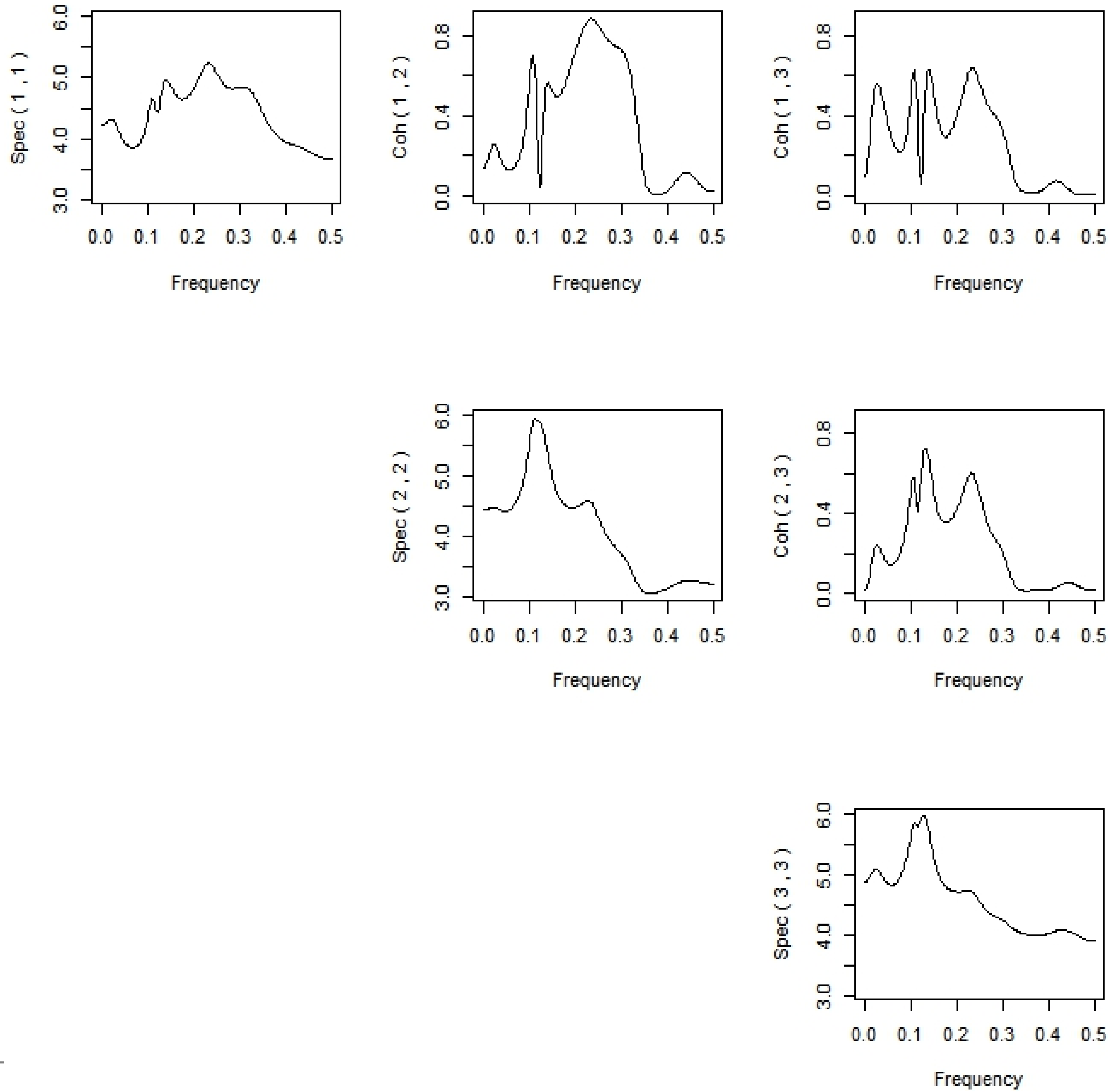
機械受注

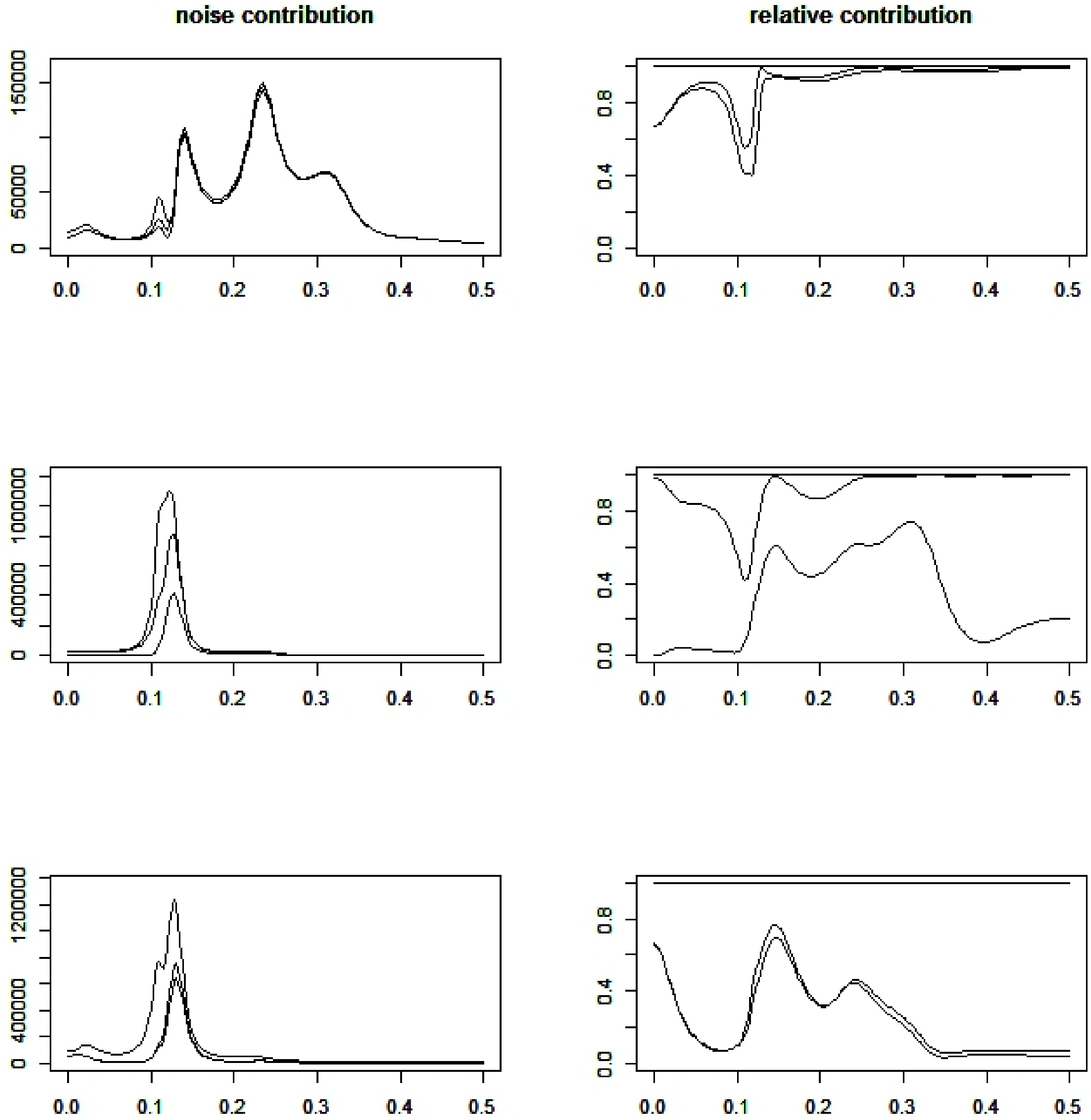


為替相場

```
data(HAKUSAN) # Yaw rate, rolling, pitching and rudder angle for the ship on the open sea
length <- dim(HAKUSAN)[1]
y <- matrix(, length/2, 3)
for( i in 1:length/2) {
	y[i,1] <- HAKUSAN[i*2,1] # yaw rate
	y[i,2] <- HAKUSAN[i*2,2] # rolling
	y[i,3] <- HAKUSAN[i*2,4] # rudder angle }
	z <- marfit(y,20)
```




Spec (1 , 1)
Coh (1 , 2)
Coh (1 , 3)
Spec (2 , 2)
Coh (2 , 3)
Spec (3 , 3)
Frequency
0.0 0.1 0.2 0.3 0.4 0.5
3.0 4.0 5.0 6.0
0.0 0.4 0.8


noise contribution
relative contribution
150000
50000
0
1000000
400000
1200000
0.0
0.4
0.8
0.1
0.2
0.3
0.5