時系列解析（2）

配布資料（2018/4/15）

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講義の概要（1）

(0) 波形を見る・・・・時系列プロット（前回）

(1) 相関をみる・・・・相関関数

(2) 周期性をみる・・・スペクトル

（１）共分散関数と相関関数

１．定常性（弱定常性と強定常性）

２．定常時系列の共分散関数

2.1　自己共分散関数の定義

2.2　自己共分散関数の性質

2.3　白色雑音

2.4　自己相関関数

2.5　多変量時系列の相互共分散関数

講義の概要（2）

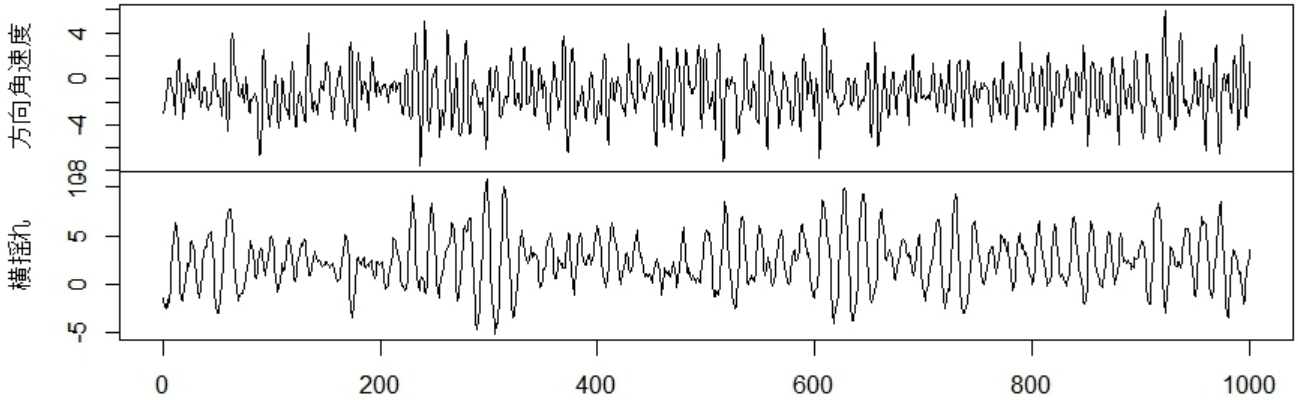
３．標本共分散関数

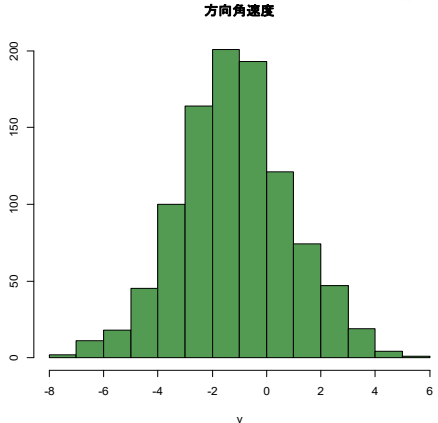
3.1　時系列における推定

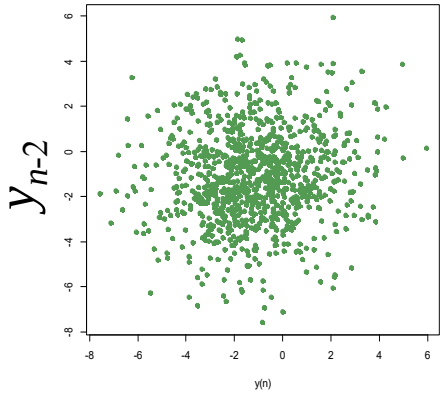
3.2　自己共分散関数の推定

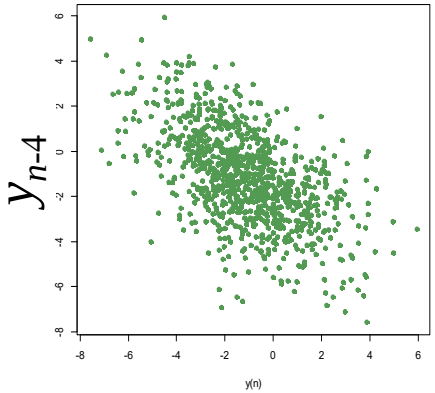
3.3　推定量の性質

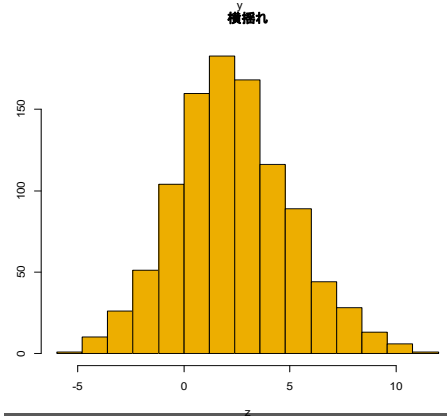
```
hist(hakusan[,1],main="方向角速度",col="#539952")
hist(hakusan[,3],main="横揺れ",breaks=seq(-6,12,1.2), col="#edae00")
```

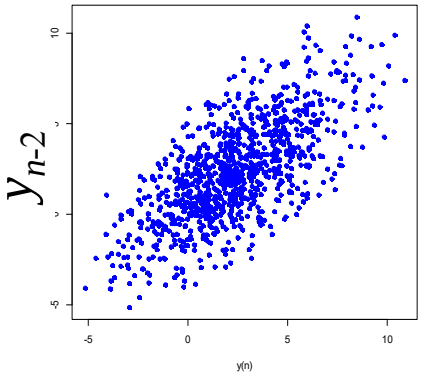


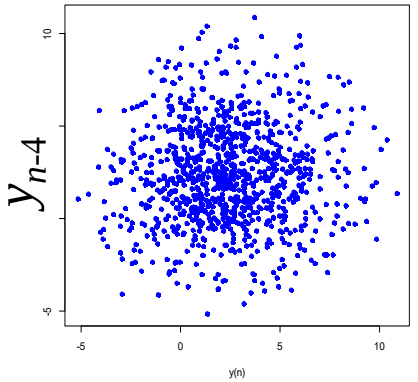












推定量の性質(白色雑音の場合)

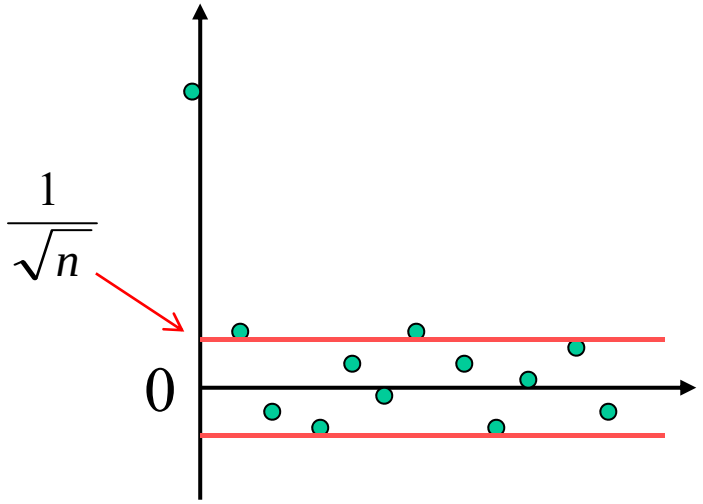
$$C_0 = \sigma^2, \quad R_0 = 1, \quad C_k = R_k = 0, \ n:\text{データ数}$$

$$\hat{\mu} \sim N(0, \frac{\sigma^2}{n})$$

$$\hat{C}_0 \sim N(0, \frac{2\sigma^4}{n})$$

$$\hat{C}_k \sim N(0, \frac{\sigma^4}{n})$$

$$\hat{R}_k \sim N(0, \frac{1}{n})$$



n	25	100	1000
$\frac{1}{\sqrt{n}}$	0.2	0.1	0.03

- 通常の誤差評価に用いられる

推定量の性質(一般の場合)

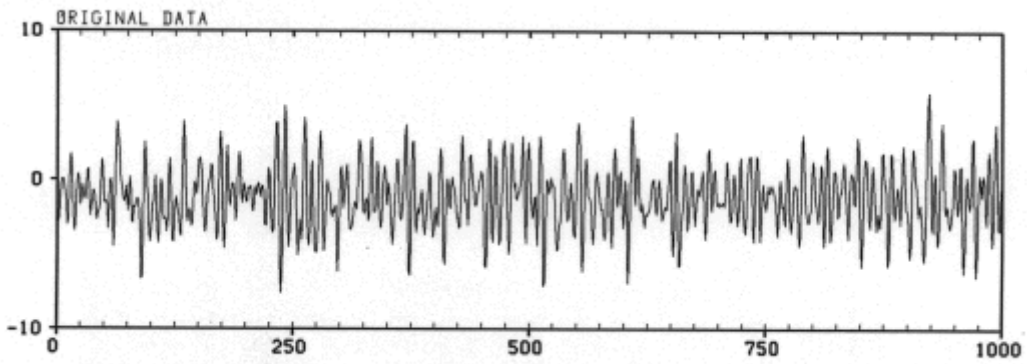
$$\hat{\mu} \sim N(\mu, \frac{1}{n}\sum_{k=-\infty}^{\infty} C_k)$$

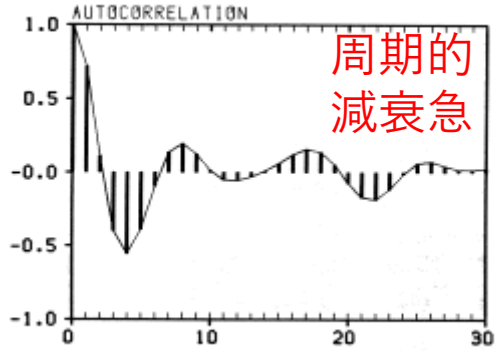
$$E[\hat{C}_k] = \frac{n-k}{n} C_k$$

$$Var(\hat{C}_k) \sim \frac{1}{n}\sum_{j=-\infty}^{\infty} (C_j^2 + C_{j-k}C_{j+k})$$

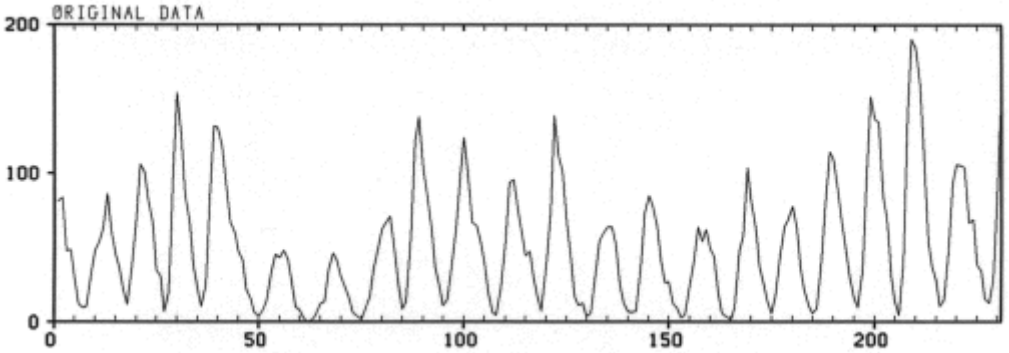
$$Var(\hat{R}_k) \sim \frac{1}{n}\sum_{j=-\infty}^{\infty} (R_j^2 + R_{j-k}R_{j+k} - 4R_jR_kR_{j+k} + 2R_j^2R_k^2)$$

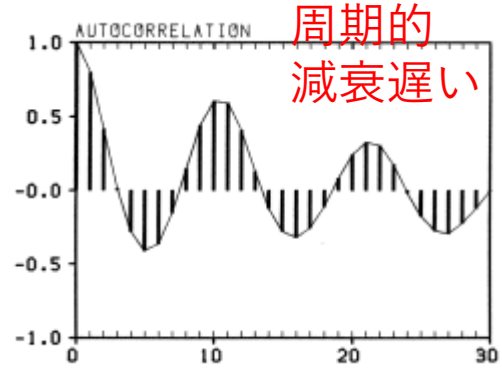
自己相関関数



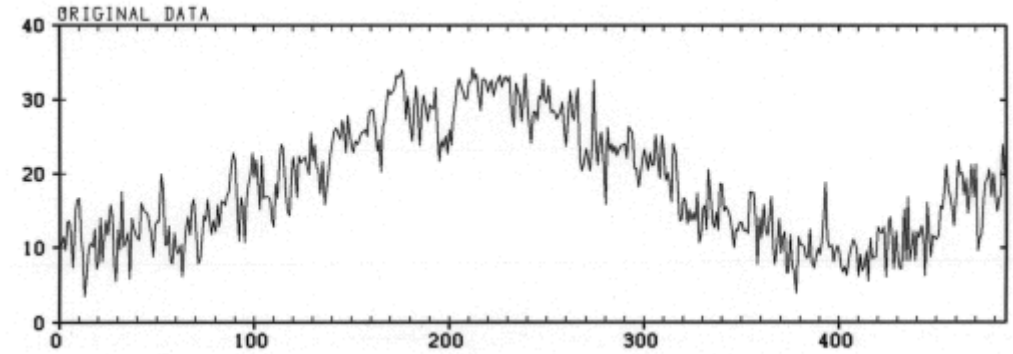


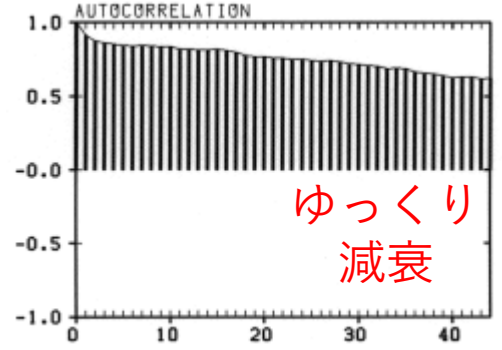
周期的
減衰急



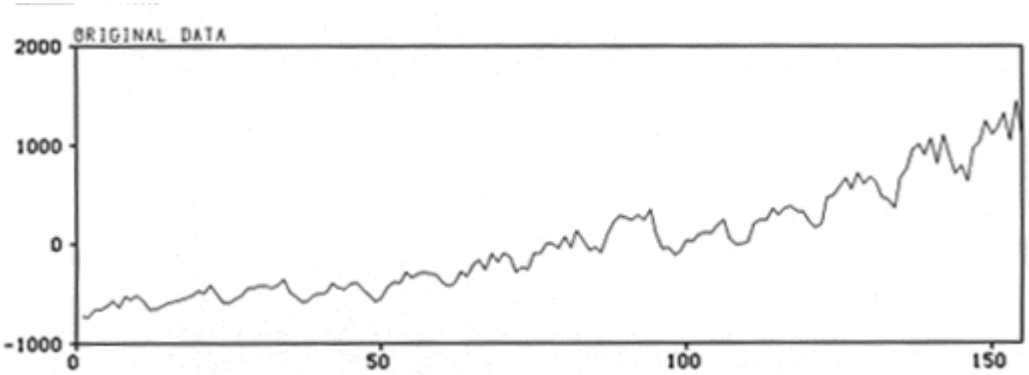


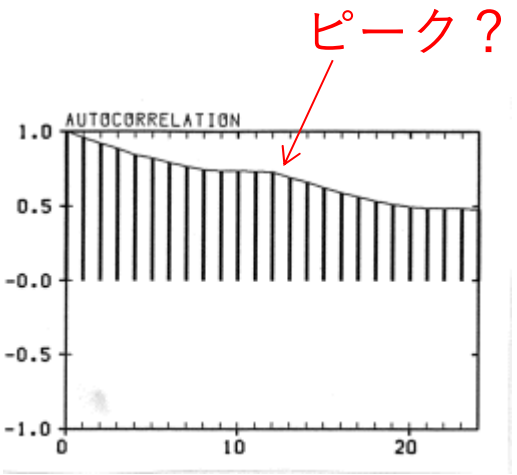
周期的
減衰遅い

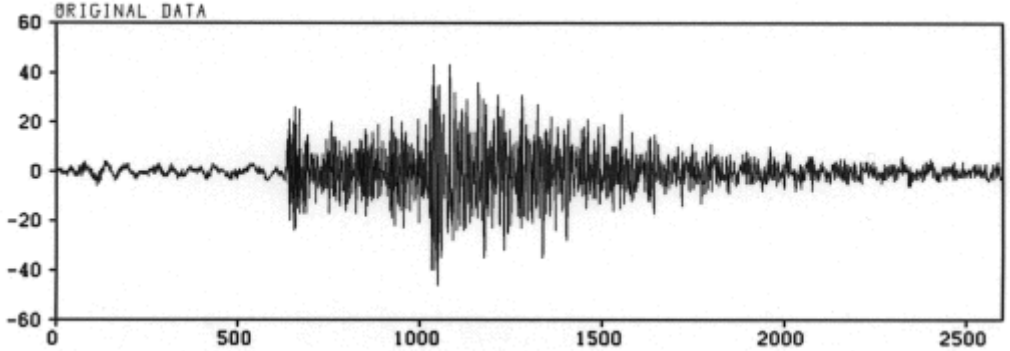


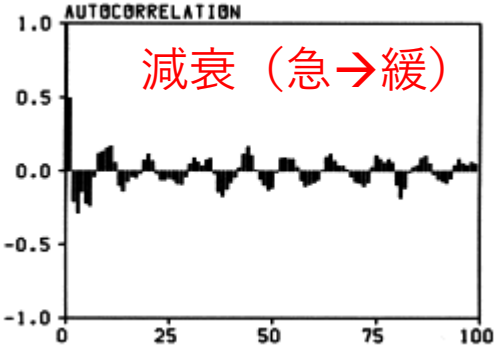


ゆっくり
減衰

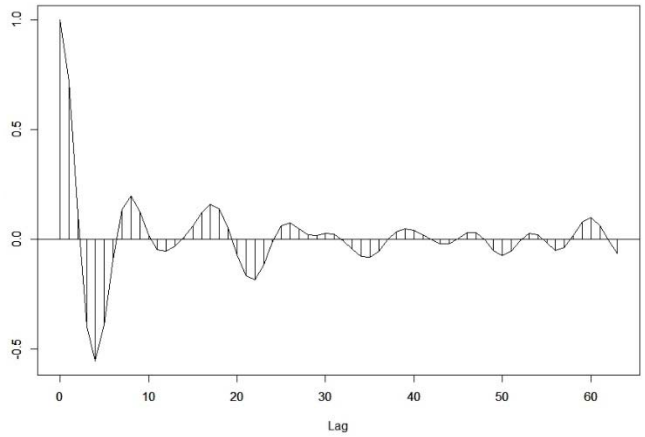

ORIGINAL DATA
2000
1000
0
-1000
0
50
100
150

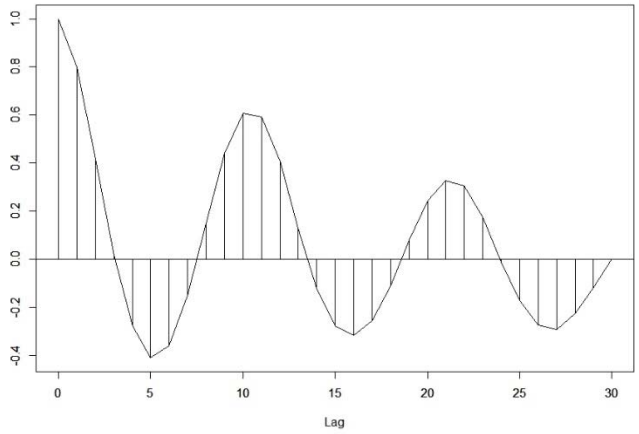

ピーク？
AUTOCORRELATION
1.0
0.5
-0.0
-0.5
-1.0
0
10
20

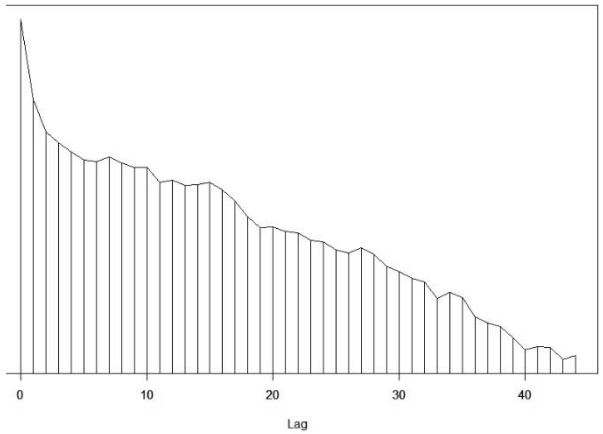

ORIGINAL DATA
60
40
20
0
-20
-40
-60
0
500
1000
1500
2000
2500

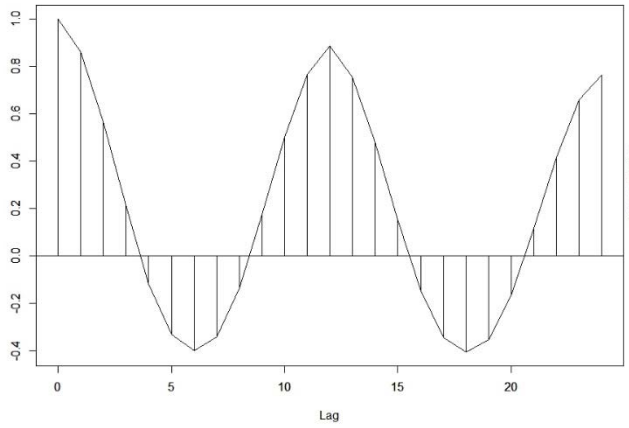

AUTOCORRELATION
減衰（急→緩）
1.0
0.5
0.0
-0.5
-1.0
0
25
50
75
100

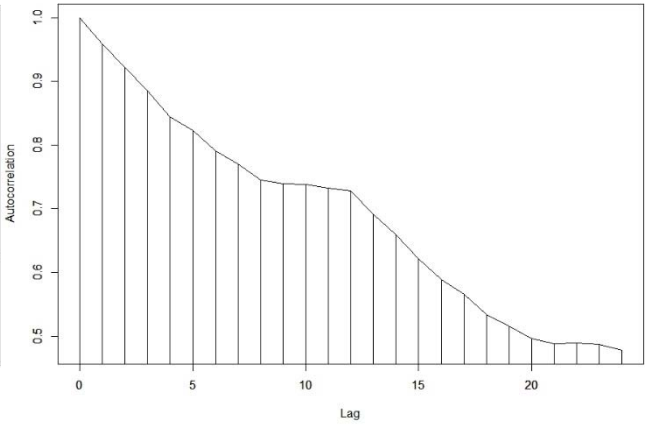
unicor(hakusan1)

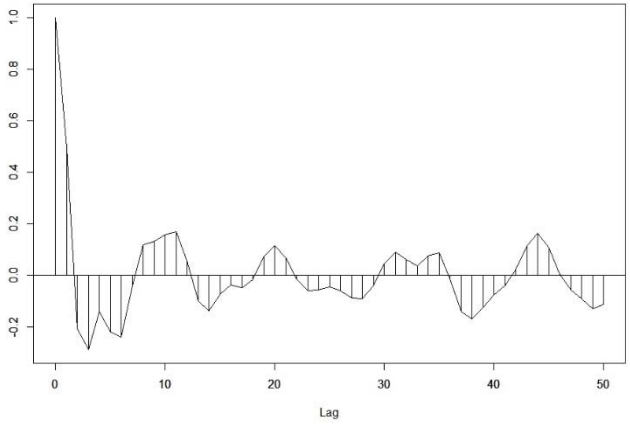




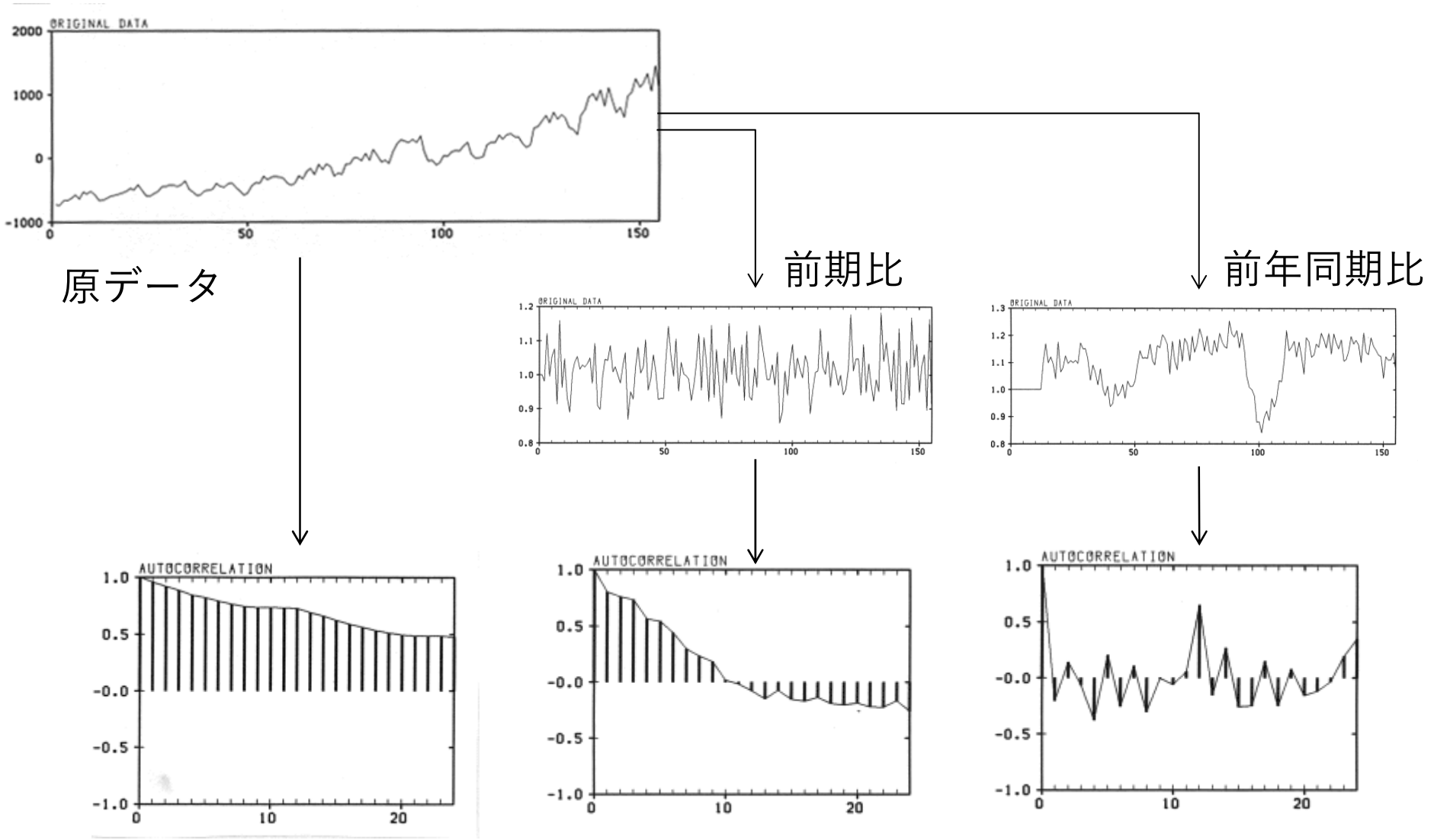




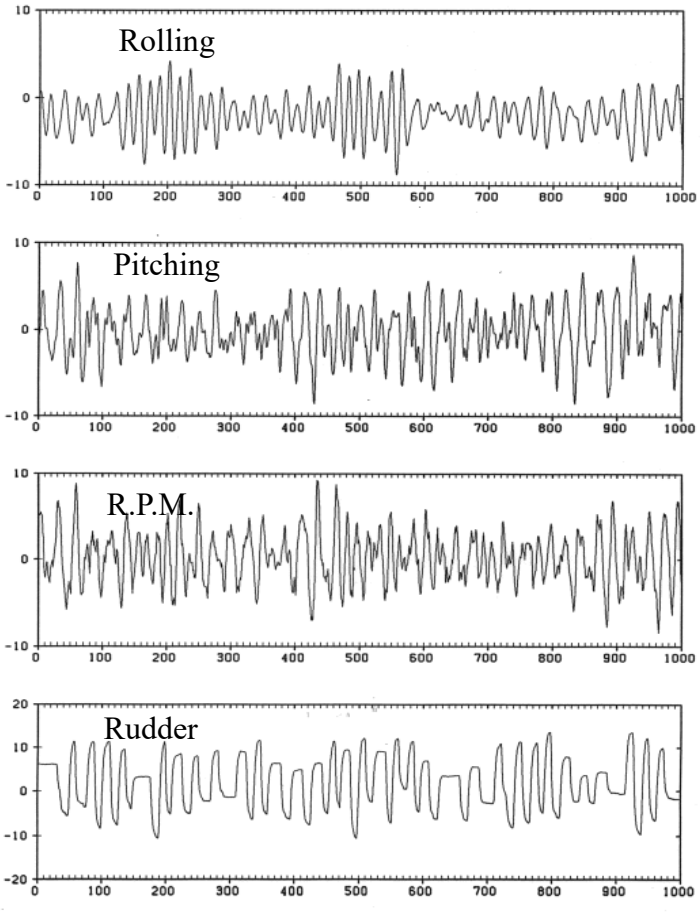


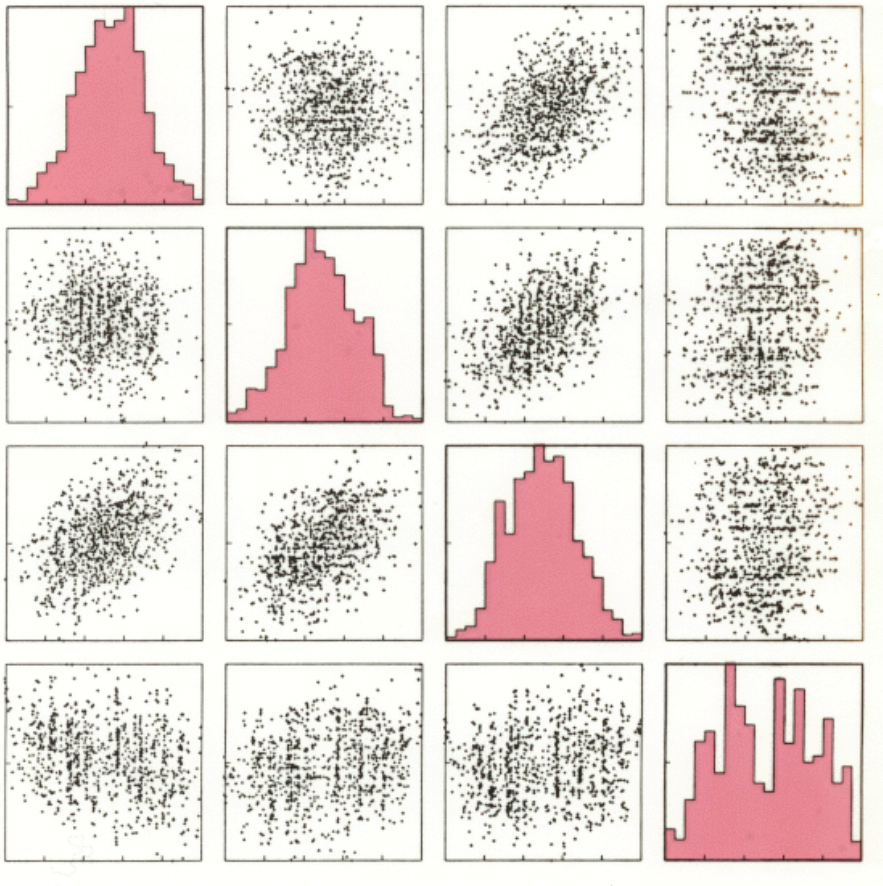


WHARD データ

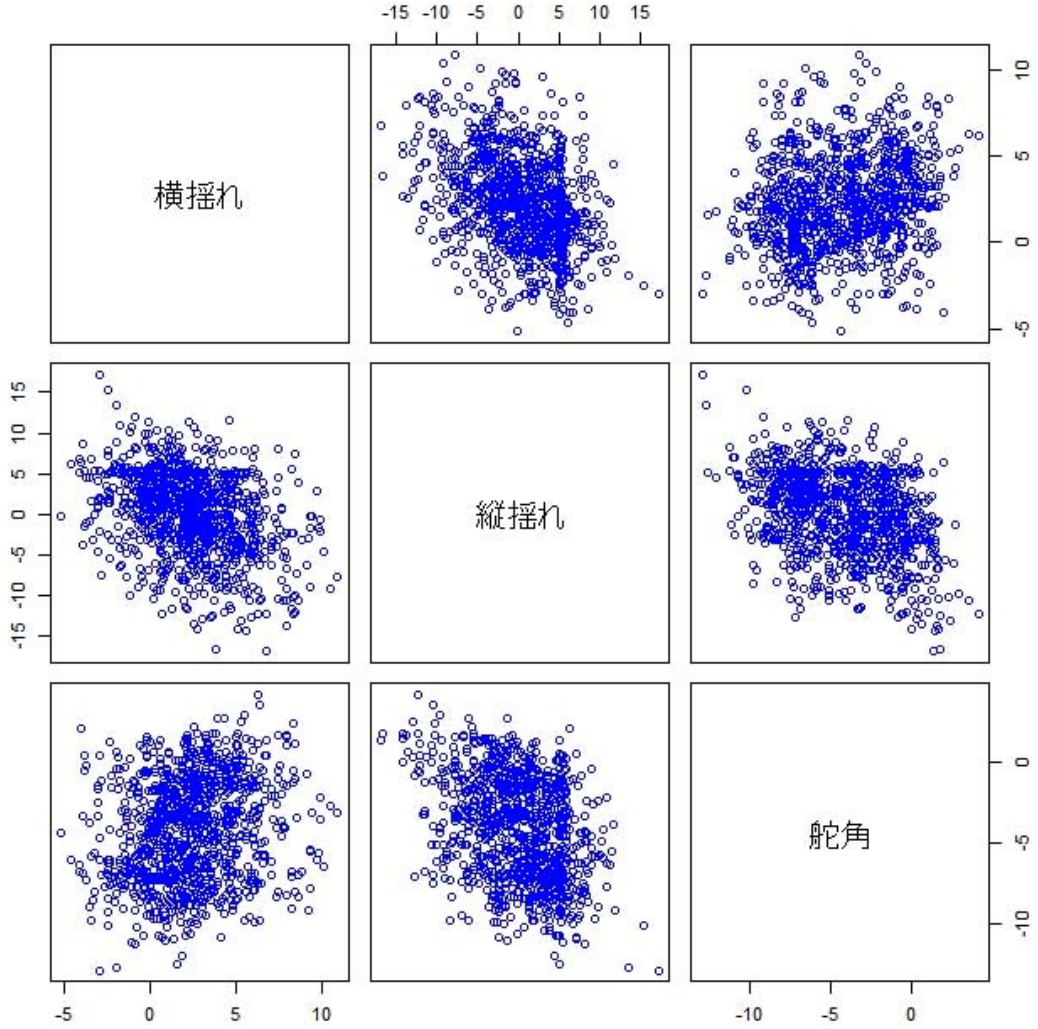

ORIGINAL DATA
2000
1000
0
-1000
0
50
100
150
原データ
前期比
前年同期比
ORIGINAL DATA
1.2
1.1
1.0
0.9
0.8
ORIGINAL DATA
1.3
1.2
1.1
1.0
0.9
0.8
AUTOCORRELATION
1.0
0.5
-0.0
-0.5
-1.0
0
10
20
AUTOCORRELATION
AUTOCORRELATION

多変量時系列



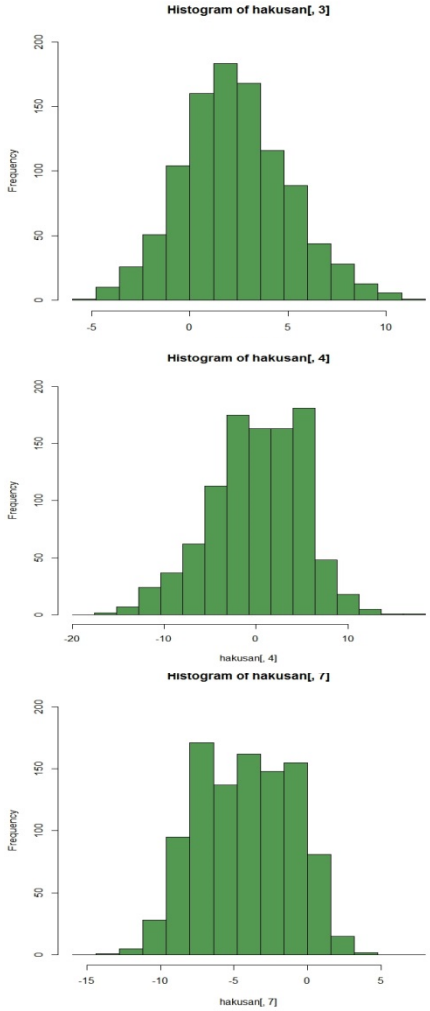


```
hakusan <- read.csv("hakusan_new.csv")
pairs(hakusan[,c(3,4,7)],col="blue")
```

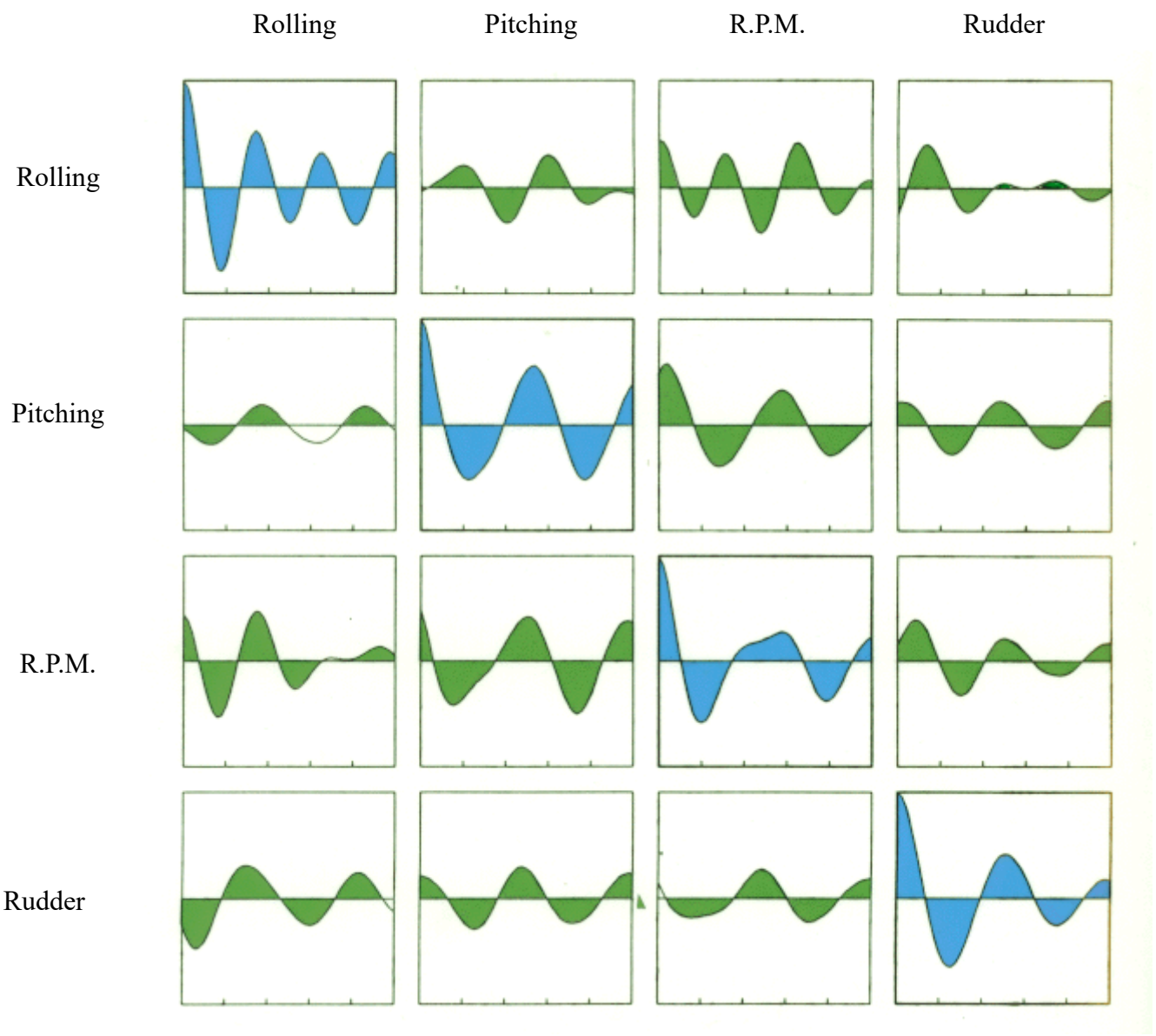



```
cor(hakusan[,c(3,4,7)])
```

	横揺れ	縦揺れ	舵角
横揺れ	1.0000000	-0.3646641	0.190980
縦揺れ	-0.3646641	1.0000000	-0.371911
舵角	0.1909800	-0.3719110	1.000000

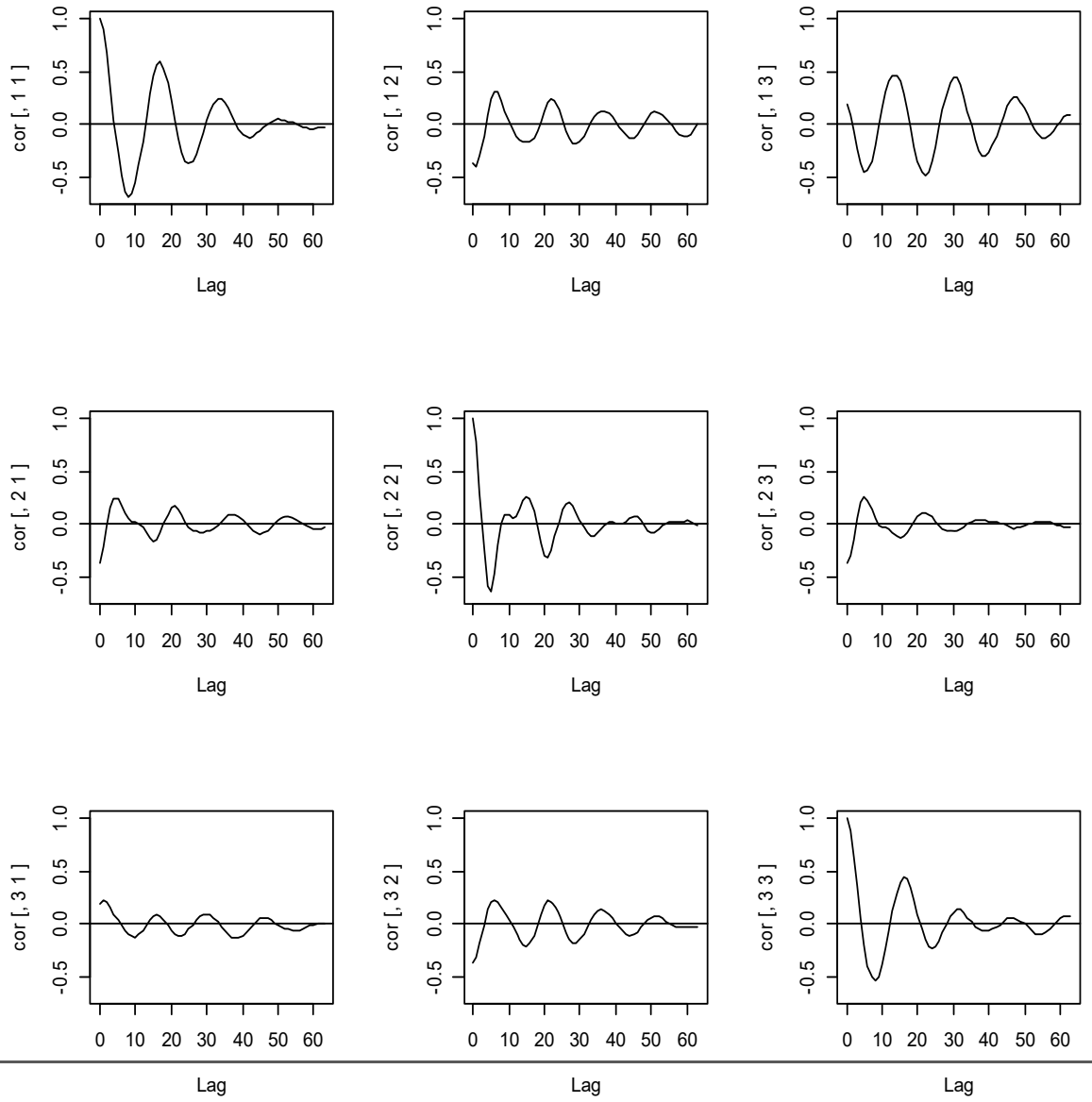


相互共分散関数



船舶データの相互相関関数

#Package(TSSS)
crscor(hakusan[,c(3,4,7)])



講義の概要（３）

（２）スペクトルとピリオドグラム

1.1　パワースペクトル

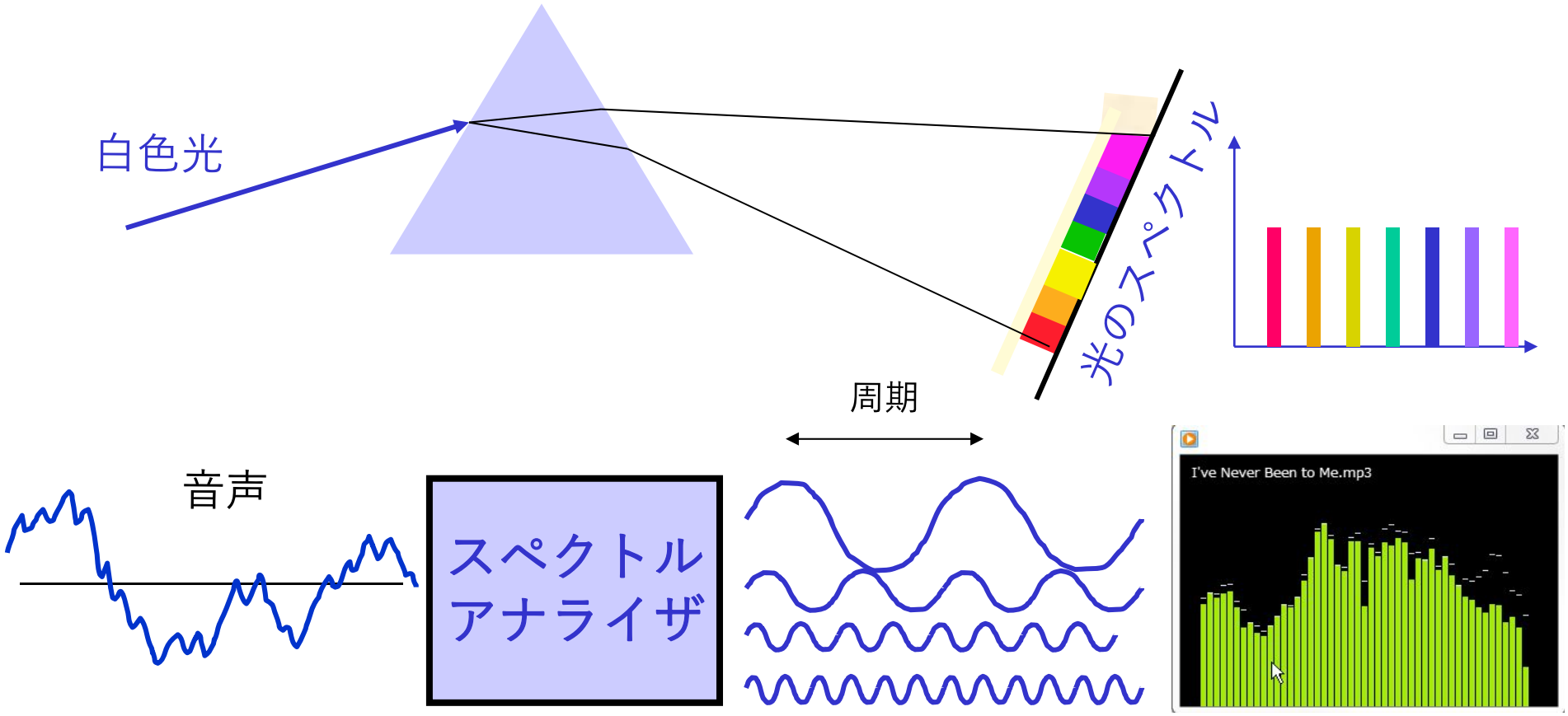
・白色雑音、ARモデル

1.2　ピリオドグラム

・ピリオドグラムの性質、平均、平滑化

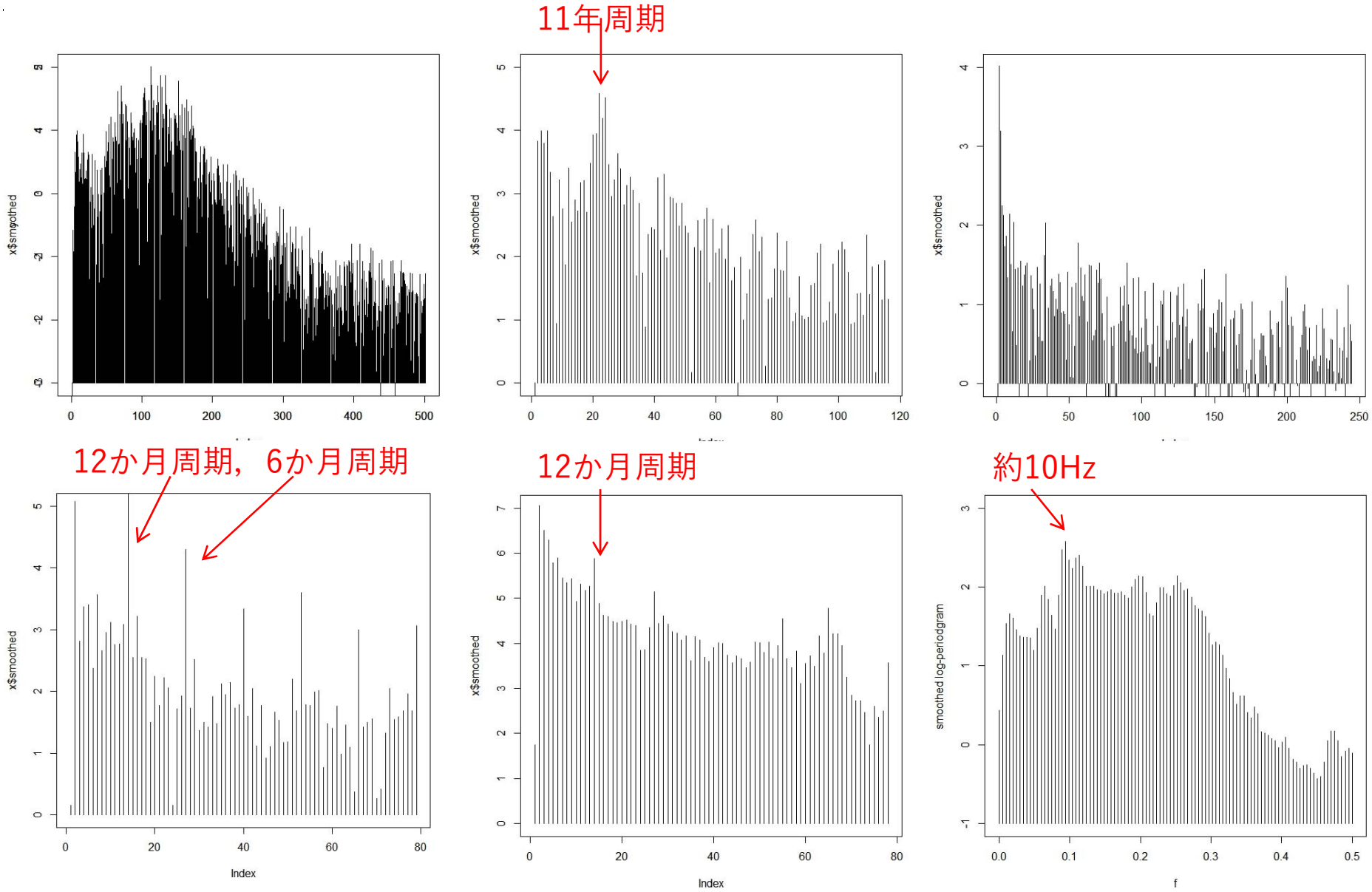
・FFT

スペクトル分解

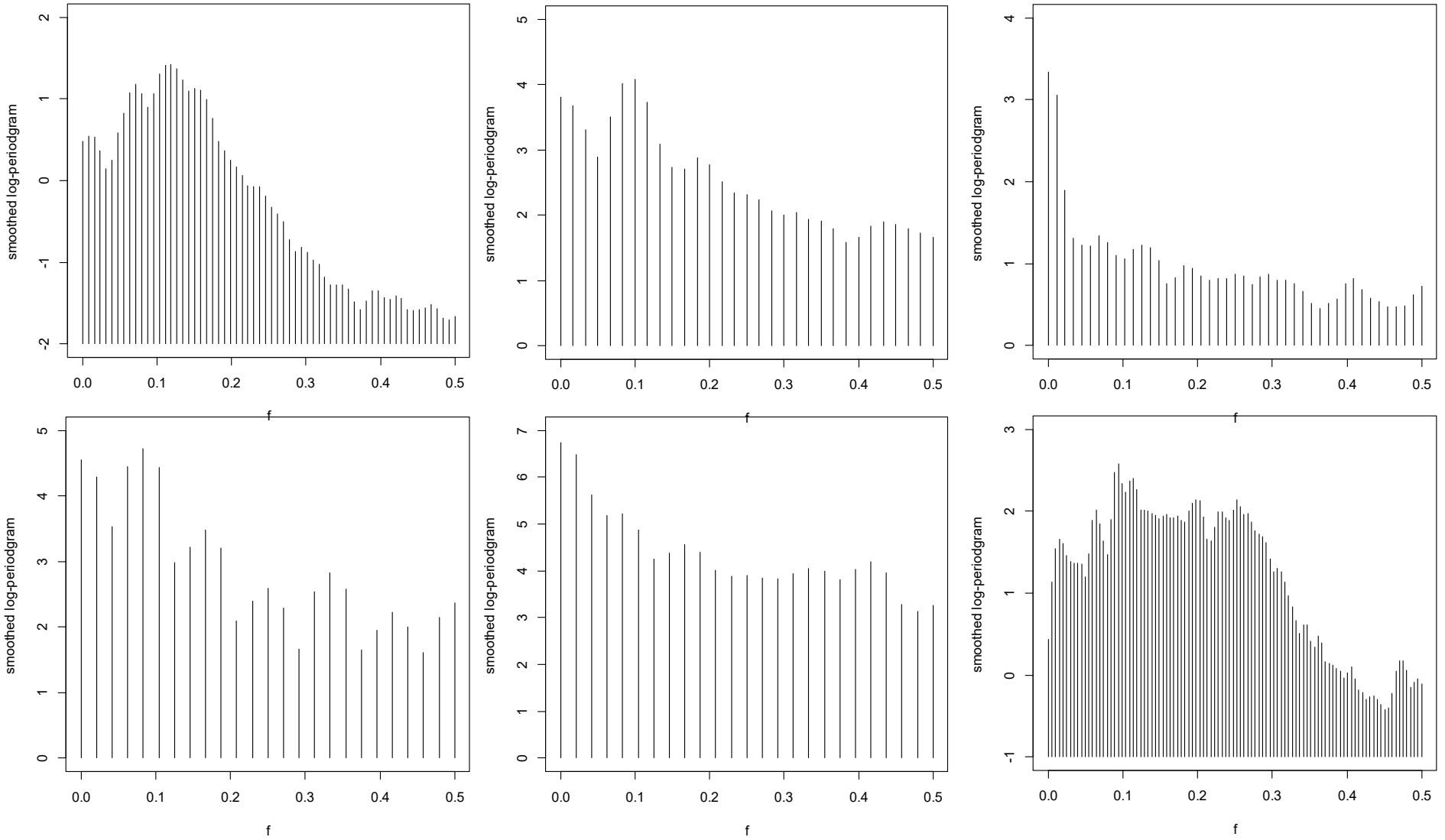


周波数=1/ 周期

ピリオドグラム

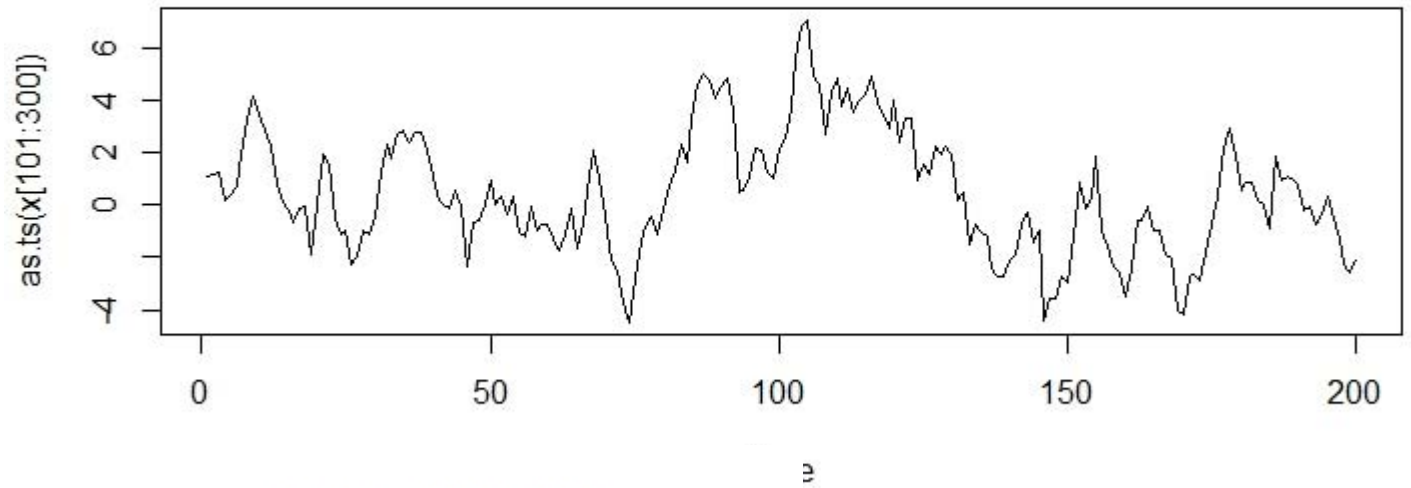

11年周期
12か月周期，6か月周期
12か月周期
約10Hz
x$smoothed
Index
smoothed log-periodgram
f

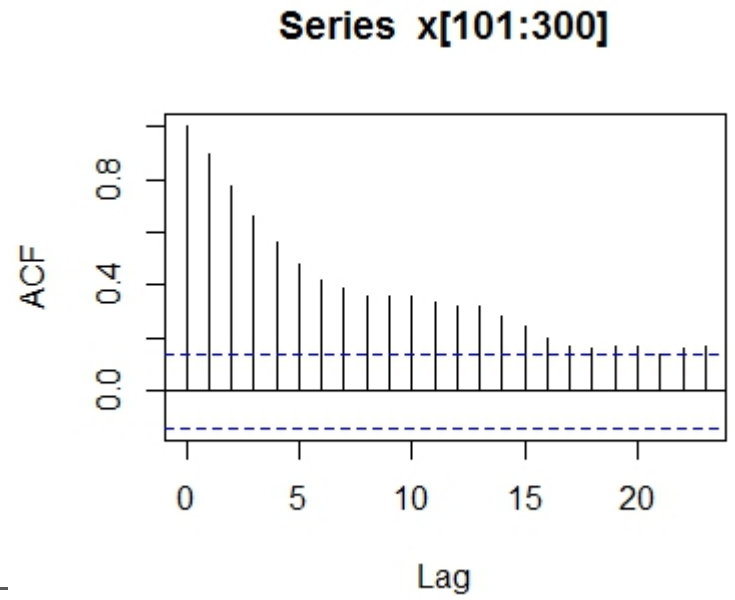
Period(hakusan1) **period(sunspot)** **period(maxtemp)**

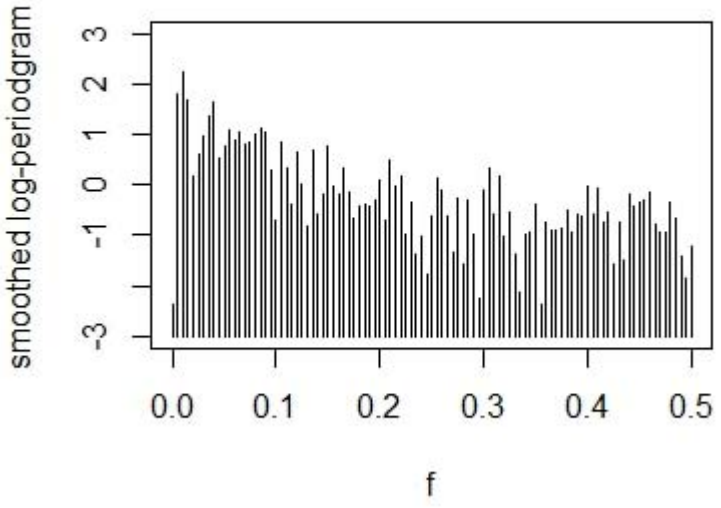


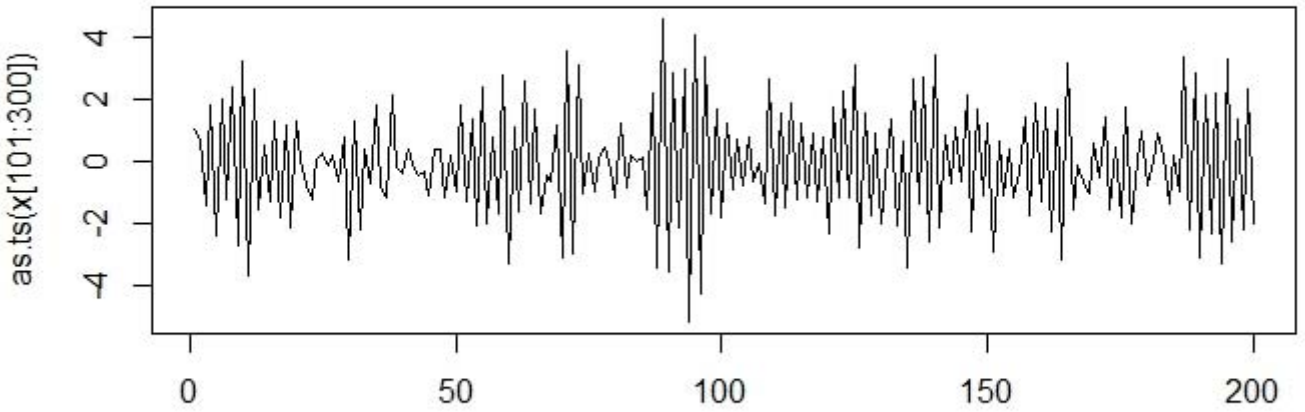

```
hakusan <- as.ts(read.csv("hakusan_new.csv"))
hakusan1 <- hakusan[,1]
sunspot <- as.ts(read.csv("sunspot_new.csv"))
maxtemp <- as.ts(read.csv("maxtemp.csv"))
blsfood <- as.ts(read.csv("blsfood_new.csv"))
whard <- as.ts(read.csv("whard_new.csv"))
 mye1f <- as.ts(read.csv("mye1f_new.csv"))
```

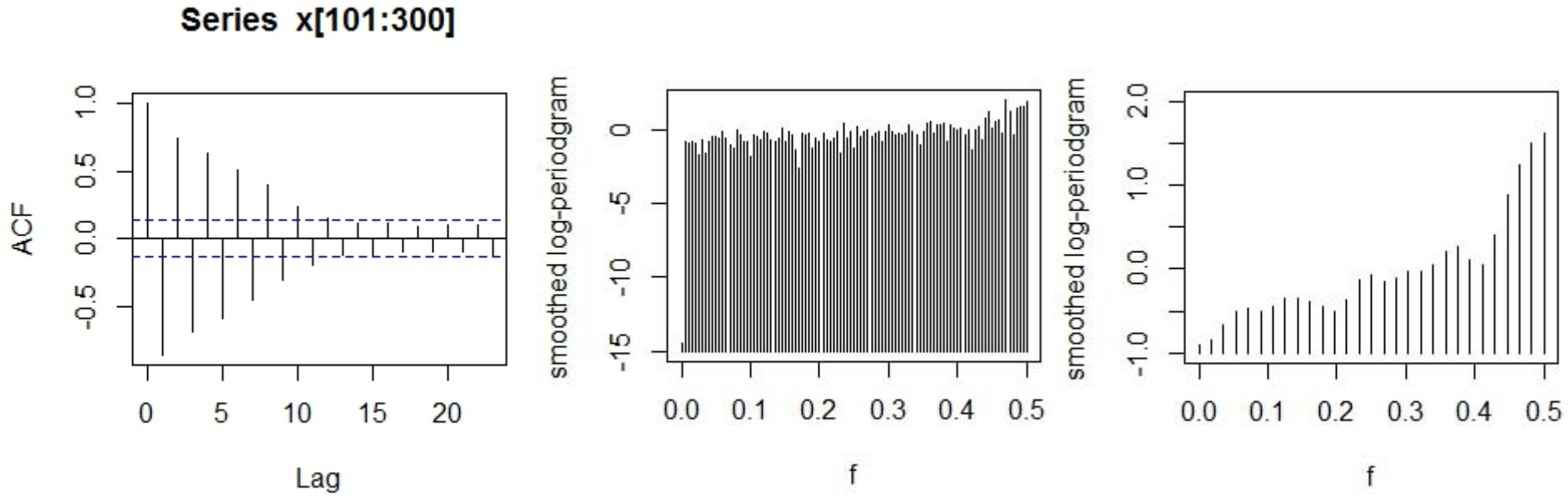
```
t <- 3:300
r <- as.ts(rnorm(300))
for (i in t)  x[i] = 0.9*x[i-1] + r[i]
plot(as.ts(x[101:300]))
```

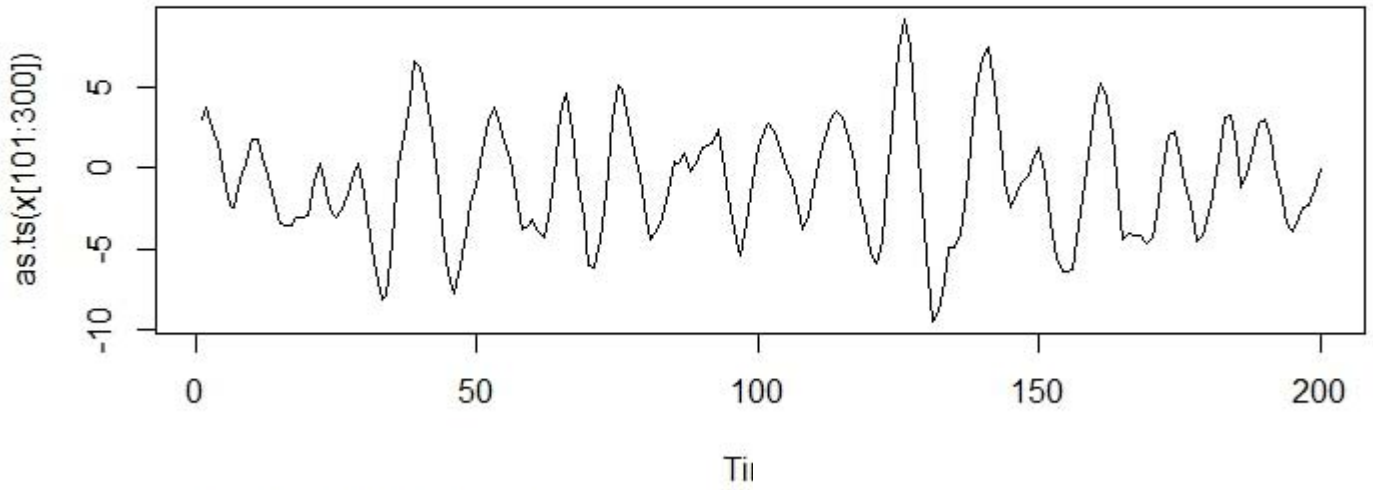


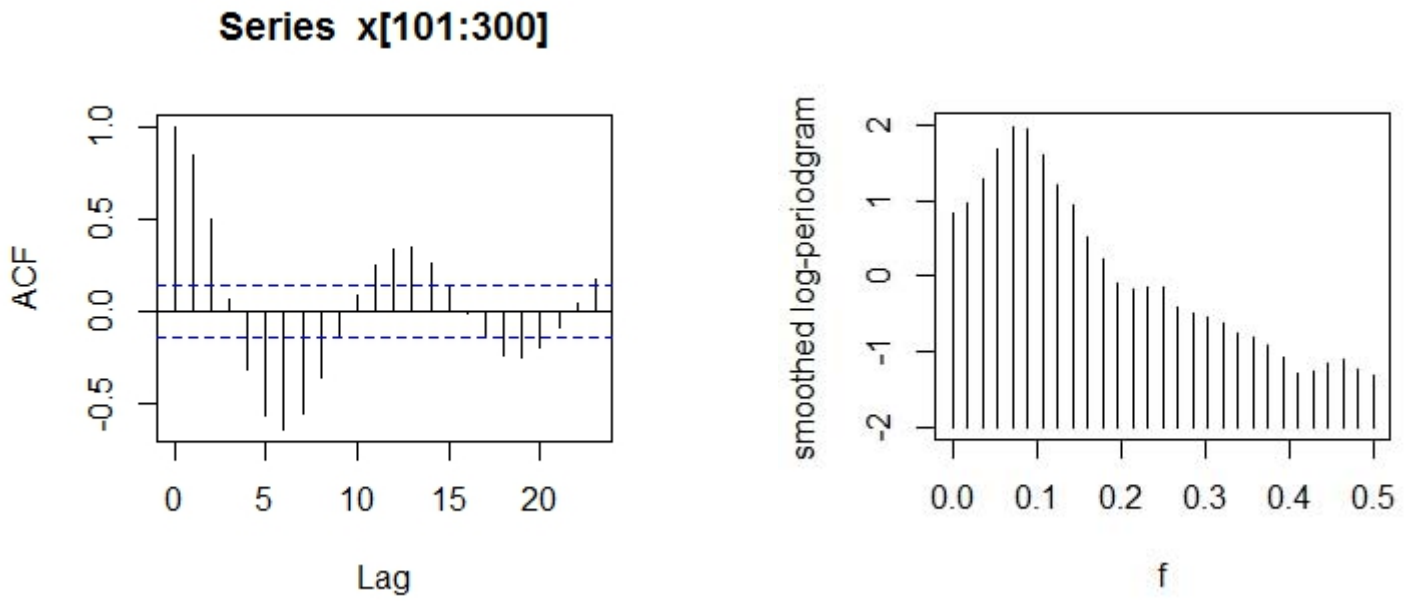




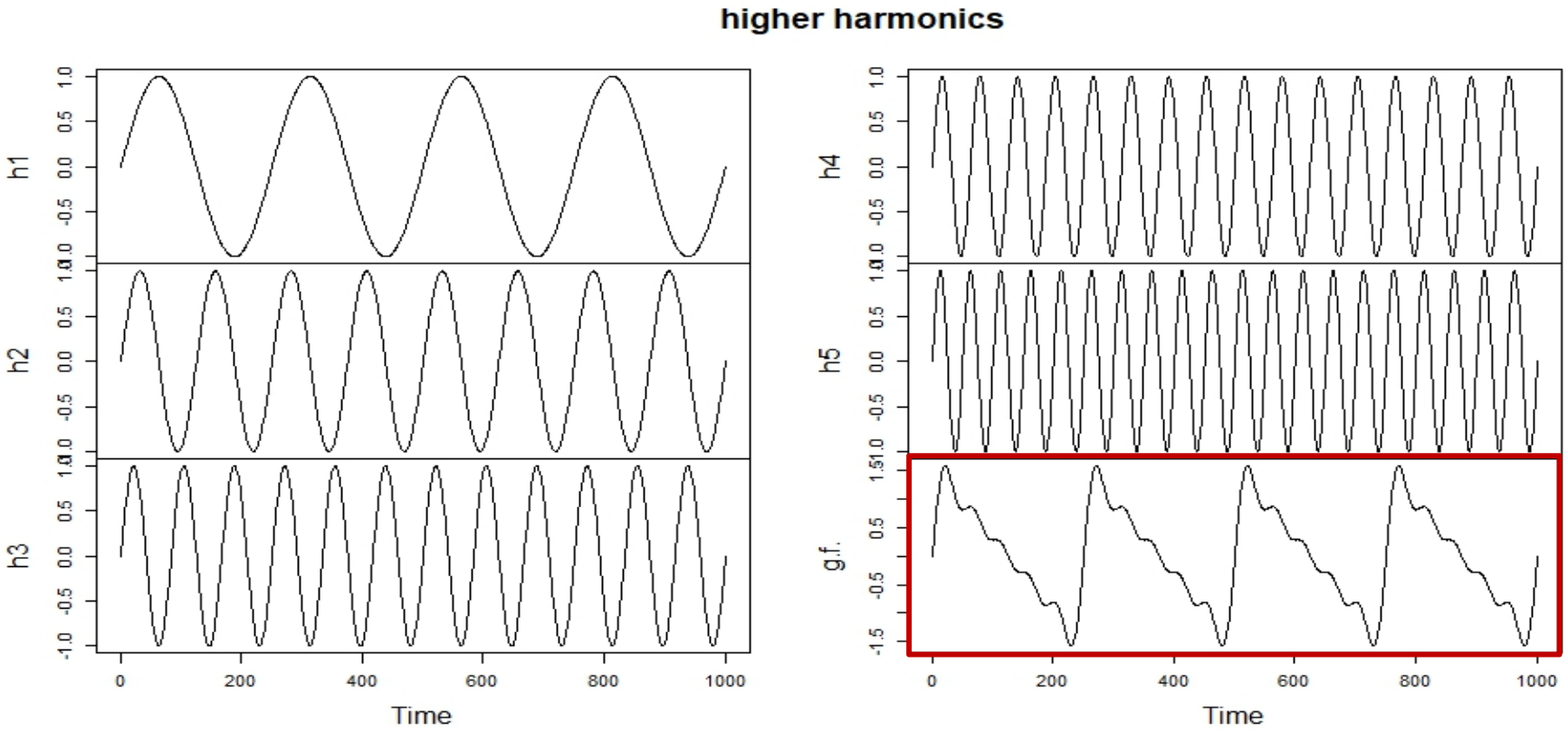

as.ts(x[101:300])
4
2
0
-2
-4
0
50
100
150
200


Series x[101:300]
ACF
1.0
0.5
0.0
-0.5
0
5
10
15
20
Lag
smoothed log-periodgram
0
-5
-10
-15
0.0
0.1
0.2
0.3
0.4
0.5
f
smoothed log-periodgram
2.0
1.0
0.0
-1.0
0.0
0.1
0.2
0.3
0.4
0.5
f

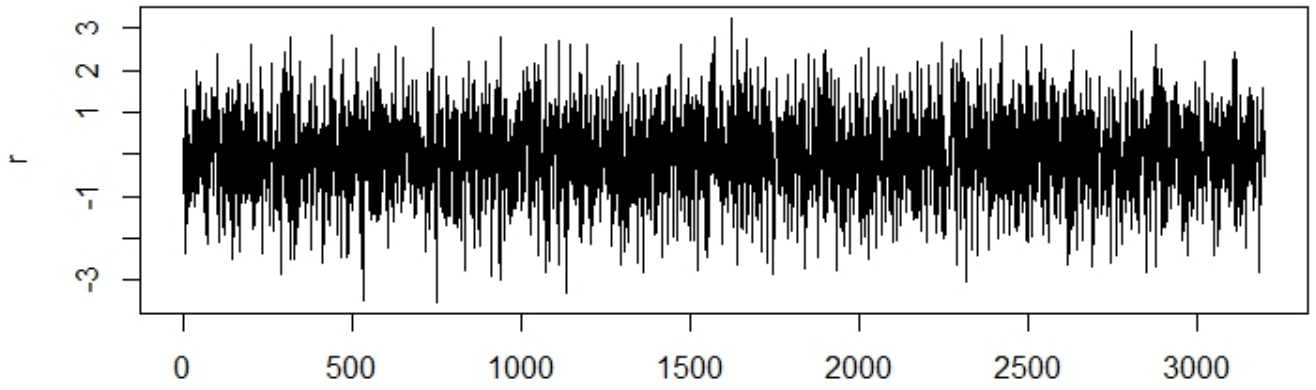

as.ts(x[101:300])
-10
-5
0
5
0
50
100
150
200
Ti

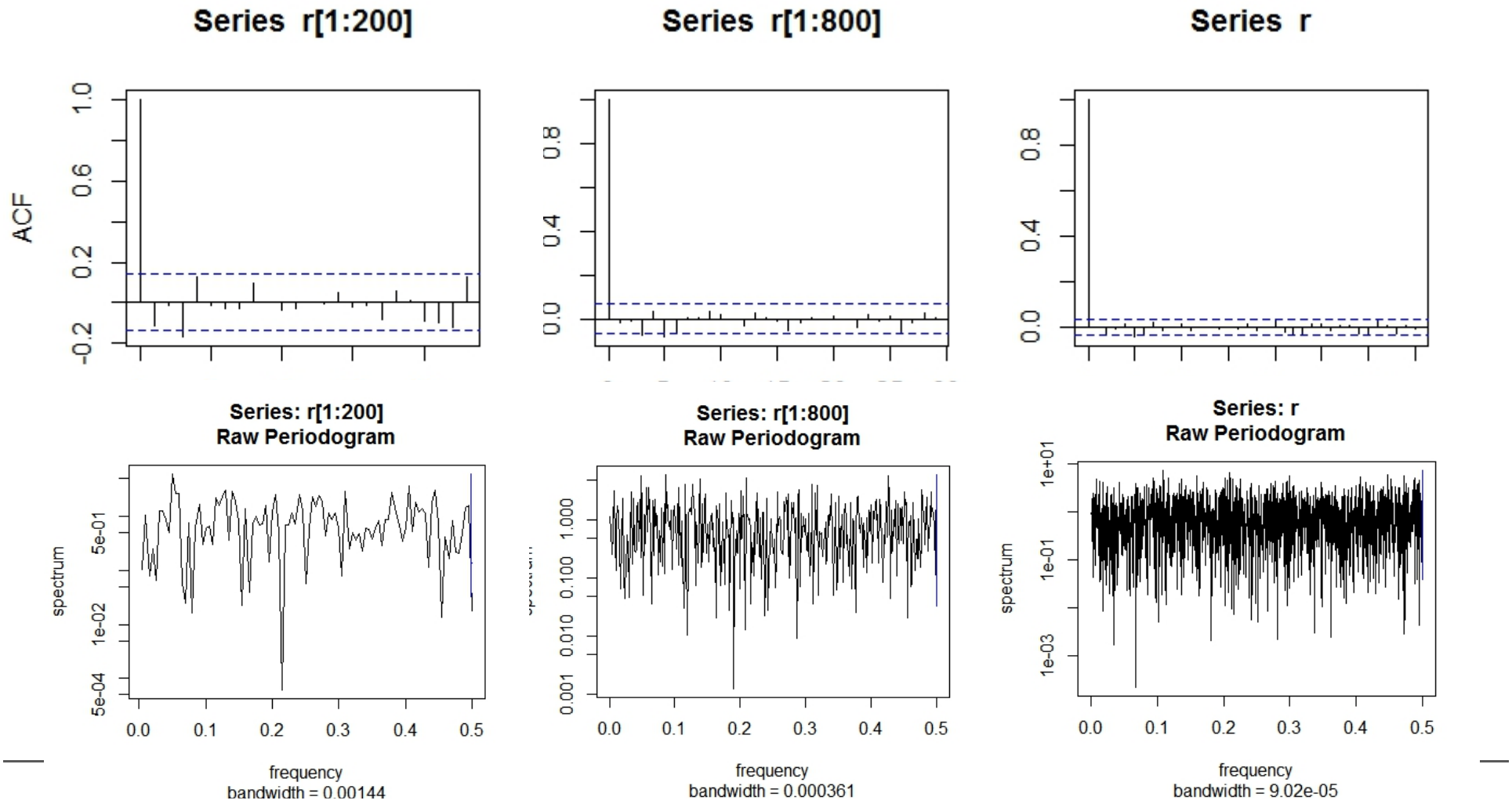

Series x[101:300]
ACF
-0.5
0.0
0.5
1.0
0
5
10
15
20
Lag
smoothed log-periodgram
-2
-1
0
1
2
0.0
0.1
0.2
0.3
0.4
0.5
f

高調波の例

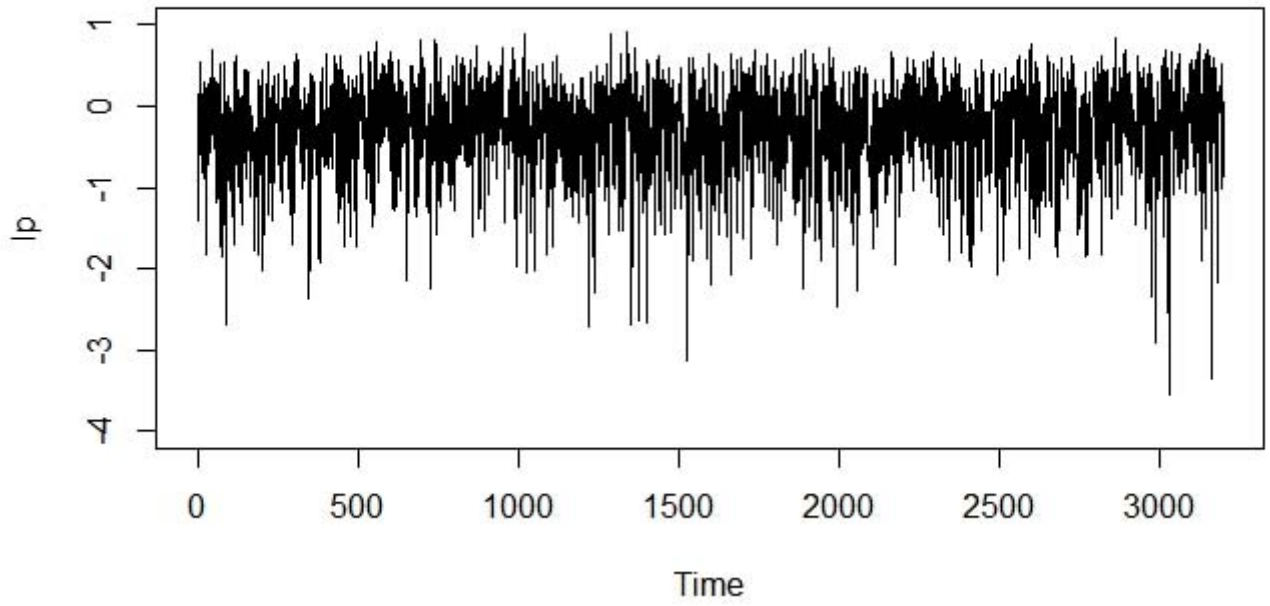

higher harmonics
h1
h2
h3
h4
h5
g.f.
Time
Time
基本波
第 2 高調波（倍音）
第 3 高調波

```
r <- rnorm(3200)
```

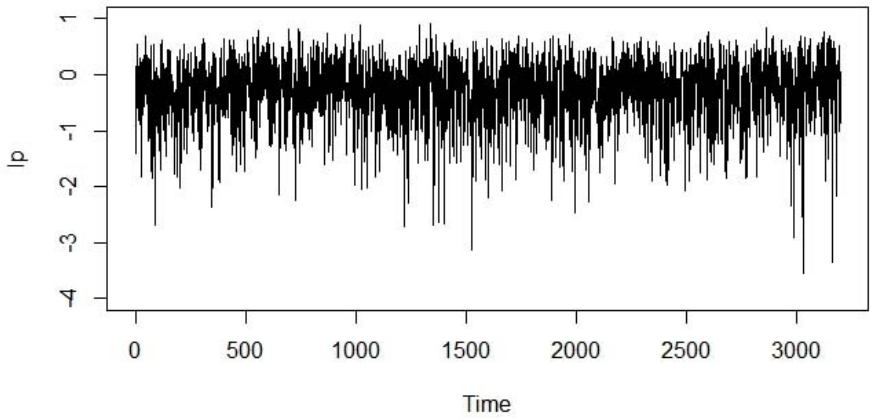

r
3
2
1
-1
-3
0
500
1000
1500
2000
2500
3000


Series r[1:200]
Series r[1:800]
Series r
ACF
1.0
0.6
0.2
-0.2
0.8
0.4
0.0
Series: r[1:200]
Raw Periodogram
Series: r[1:800]
Raw Periodogram
Series: r
Raw Periodogram
spectrum
5e-01
1e-02
5e-04
1.000
0.100
0.010
0.001
1e+01
1e-01
1e-03
0.0
0.1
0.2
0.3
0.4
0.5
frequency
bandwidth = 0.00144
frequency
bandwidth = 0.000361
frequency
bandwidth = 9.02e-05

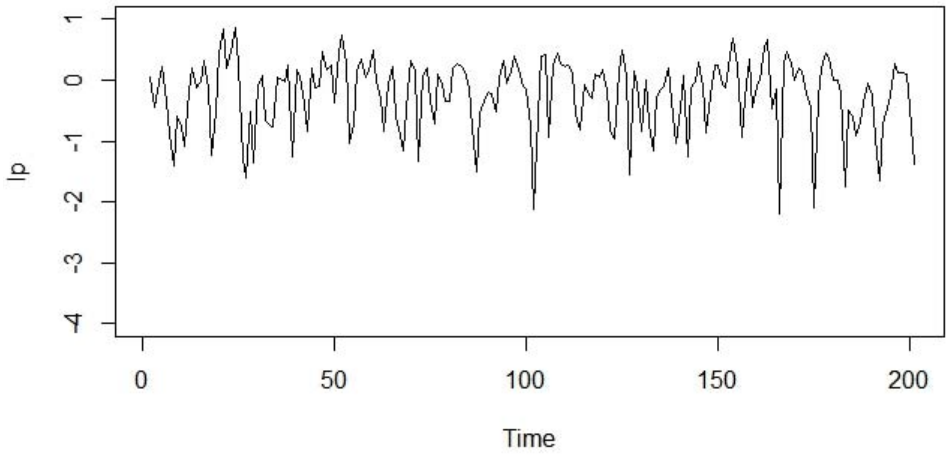
```
##  Raw Spectrum
# autocovariance function
lag <- 50
lag <- length(r)
lag1 <- lag-1
x <-
as.ts(acf(r,type="covariance",lag.max=lag))
c <- x$acf

# raw spectrum
p <- rep(1:lag)
f <- 0:lag
g <- 1:lag1
for (i in f)  {
  fj <- i/(2*lag)
  s <- c[1]
  for (k in g) {
     s <- s + 2*c[k+1]*cos(2*pi*k*fj)
     }
  p[i+1] <- s
}
p <- as.ts(p)
lp <- log10(p)
plot(lp,ylim=c(-4,1))
```



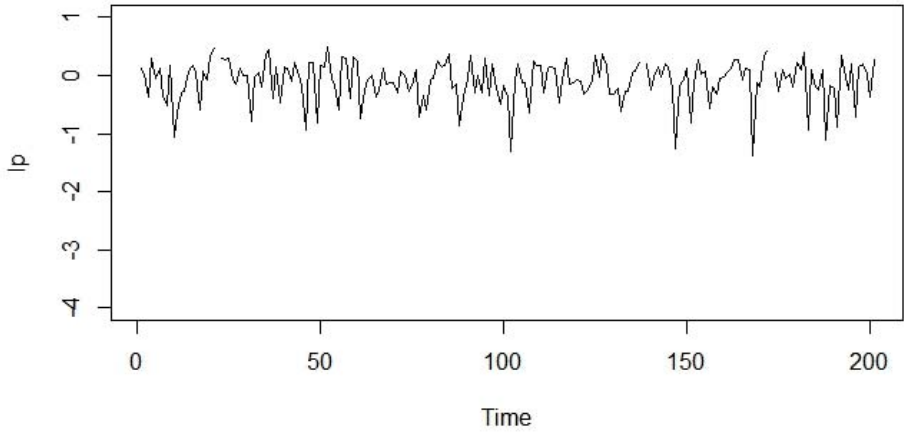
N=3200, lag=3200



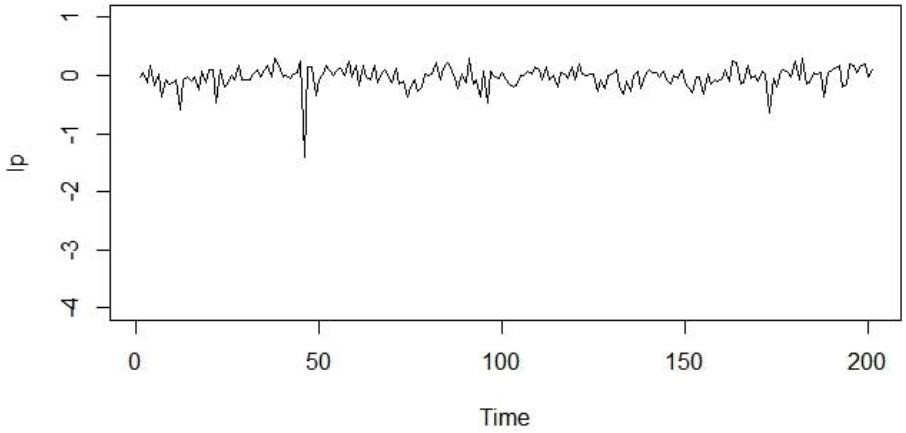
N=200, lag=200



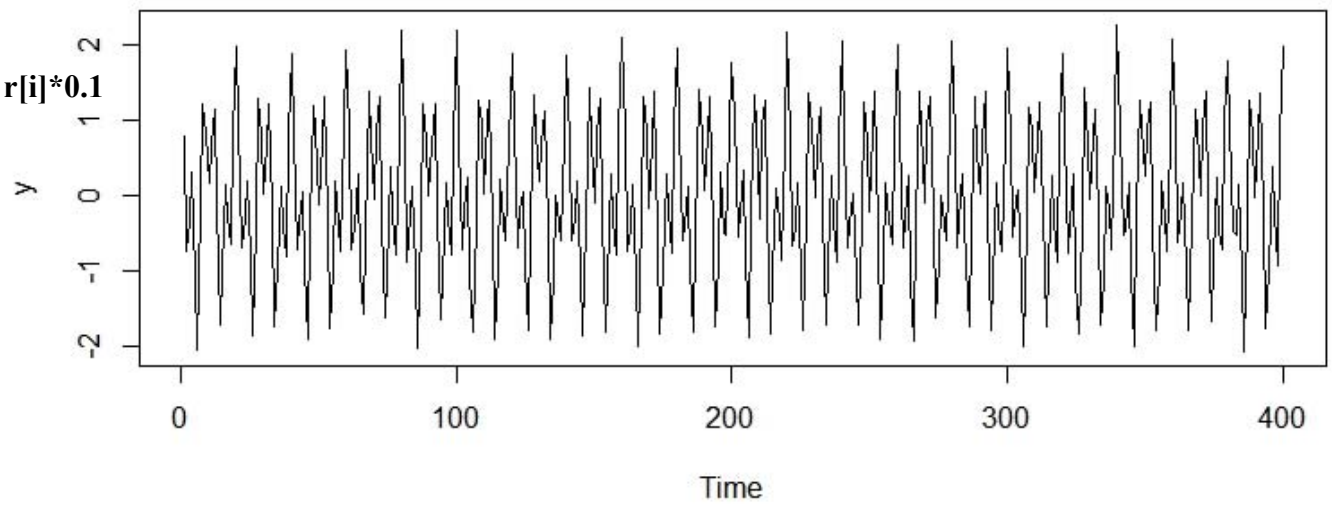
N=800, lag=200



N=3200, lag=200

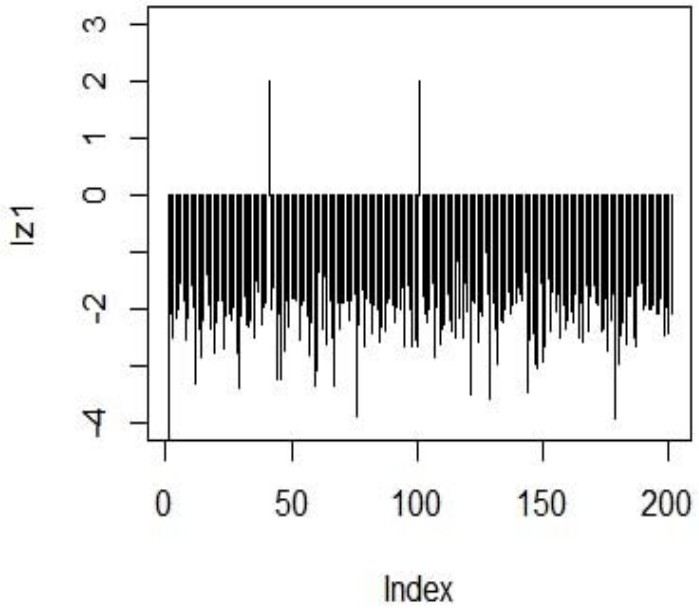


```
# simulation by 2 cosine function
t <- 1:400
r <- rnorm(400)
for (i in t) {
y[i] <- cos(2*pi*i/10) + cos(2*pi*i/4) + r[i]*0.1
}
y <- as.ts(y)
plot(y)
```

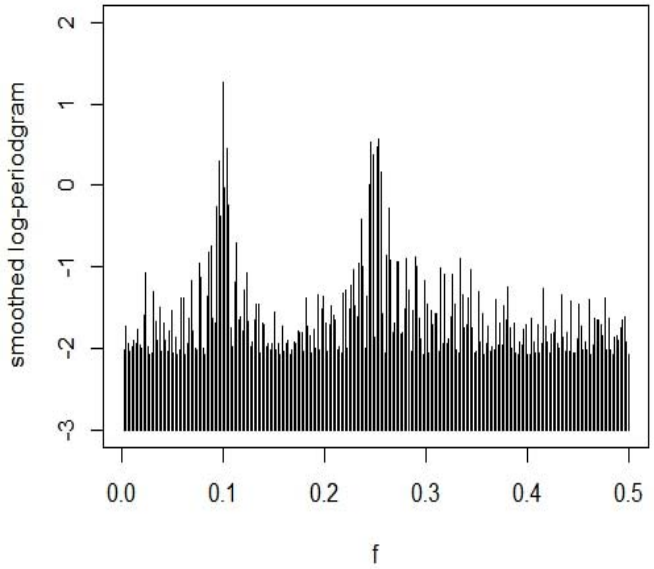



```
z <- period(y,window=0)
z1 <- z$period
lz1 <- log10(z1)
plot(lz1,type="h")
```

ピリオドグラム



FFTピリオドグラム



FFTピリオドグラム（船舶方向角データ）